Absorption in One-Dimensional Lossy Photonic Crystal

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Abstract: In this work we have obtained the absorption property in one-dimensional (1D) defective lossy photonic crystals (PCs) composed of double-negative (DNG) and double-positive materials by using transfer matrix method. The considered asymmetric geometric structures contain a defect structure at the center of the host crystal as a photonic-quantum-well (PQW) with two different types of DNG layers. This study investigates the effect of the stack number of the bilayers PQW defect on the number, position, and the rate of the absorption of anti-resonant modes for both types of the DNG defect. The results show that these factors of the anti-resonant modes depend on the types of the DNG metamaterial corresponding to the defect layers. Accordingly, by changing the types of PQW defect, the anti-resonant modes are red-shifted or blue-shifted as a function of the stack number of PQW. Furthermore, the effect of loss factors on the rate of absorption is examined for the two different types of PQW. The results can present helpful information for designing new types of narrowband and multichannel filters at the microwave.

Keywords: One-dimensional photonic crystal, Anti-resonant mode, Double-negative materials, Absorption, Photonic quantum-well.

1. INTRODUCTION

Photonic crystals (PCs) are artificial materials containing a periodic multilayer construction in dielectric constant which have some remarkable optical characteristics [1]. The most considerable property is that there exists a forbidden band in some frequency regions in a typical all-dielectric periodic structure. This forbidden band is also known as a photonic band gap (PBG) [1-5]. In recent years, the other optical properties of a PC containing the so-called double-negative (DNG) metamaterials have attracted a great deal of attention [6-10]. DNG materials are artificial composites with simultaneously negative permittivity and negative permeability and were first theoretically investigated by Veselago in 1968 [11]. The DNG metamaterial as an artificial structure was studied experimentally by Smith and his group in 2000 [12-14]. A principal feature of a DNG metamaterial is the refractive index of such material is negative. Consequently, a DNG metamaterial is also mentioned as a negative-index material (NIM). Based on the requirement of causality principle together with the Kramers-Kronig relations, NIMs are not only dispersive but also lossy [15]. With the possibility of producing metamaterials, PCs containing metamaterials known as metamaterial photonic crystals (MetaPCs) have been developed. The conventional PCs and MetaPCs because of their unique electromagnetic properties and scientific and engineering applications have attracted extensive attention by many researchers in recent years [16-30].

By interrupting the periodicity of a usual PC structure with introducing a layer with different optical properties, we will have a defective crystal and a localized defect mode will appear inside the band gap [31]. The potential applications of defective PCs in different areas, such as light emitting diodes, narrowband transmission filters, and fabrication of lasers have made such structures an interesting research topic. The defected layer added to a host PC of \((AB)^N\) can be replaced by another PC of \((CD)^N\).

The structure is presented by \((AB)^{N/2}\) \((CD)^{M}\) \((AB)^{N/2}\), where \(N\) and \(M\) are respectively the number of lattice period of the \((AB)\) and \((CD)\) bilayers. In this situation, the defect structure \((CD)^{N}\) is identified as a photonic-quantum-well (PQPW) [32-34]. Some experimental and theoretical studies have also been conducted by using the conventional and PQW defects [1, 32-45].

The main purpose of this work is to conduct a theoretical investigation of the effects of two different types of DNG defect layers on the absorption property of one-dimensional (1D) lossy MetaPC containing a PQW defect. Our study also investigates the behavior of absorption and anti-resonant mode (defect mode) by applying different values of magnetic and electric loss factors of the DNG defect structure.

2. THEORETICAL DETAILS AND CHARACTERISTIC MATRIX METHOD

The 1D MetaPC modeled as a periodic multilayer and asymmetric structure immersed in free space with
a defect structure (PQW defect) at the center of the host PC as in Figure 1. DNG metamaterials are dispersive and dissipative. Layers $A$ and $C$ are considered to be DNG metamaterials, and layer $B$ is a DPS material. Permittivity, permeability and thickness of the layers are respectively considered to be \( \varepsilon_i \), \( \mu_i \), and \( d_i \) \((i = A, B, C)\). Moreover, \( N \) and \( M \) respectively specify the number of lattice period and the number of defect structure unit cells.

The transfer matrix method [46, 47] is considered to be the most effective technique to analyze the transmission features of 1D PC structures, which is applied to perform our corresponding calculations. 

\[
M \left[ d \right] = \left( M_A M_B \right)^{N/2} \left( M_B M_C \right)^M \left( M_A M_B \right)^{N/2}
\]

is the offered transfer matrix for asymmetric \((AB)^{N/2} (CB)^M (AB)^{N/2}\) defective structures, where \(M_A\), \(M_B\), and \(M_C\) are the transfer matrices of layers \(A\), \(B\), and \(C\). The transfer matrix \(M_i\) is obtained by the following equation for the TE wave at the incidence angle \(\theta_0\) from vacuum to a 1D PC structure [46]:

\[
M_i = \begin{bmatrix}
\cos \gamma_i & -i \sin \gamma_i \\
-ip_i \sin \gamma_i & \cos \gamma_i
\end{bmatrix},
\]

where \(\gamma_i = (\omega/c) n_i d_i \cos \theta_i\), \(c\) is the speed of light in a vacuum, \(\theta_i\) is the angle of refraction inside layer \(i\) with refractive index \(n_i\) and \(p_i = \sqrt{\varepsilon_i/\mu_i} \cos \theta_i\) in which \(\cos \theta_i = \sqrt{1-n_i^2 \sin^2 \theta_i/n_0^2}\) wherein \(n_0\) is the refractive index of the environment where the multilayer structure immersed in. The refractive index is given as \(n_i = \pm \sqrt{\varepsilon_i \mu_i}\) [48,49], in which the positive sign is for the double-positive (DPS) material whereas the negative sign is for the DNG metamaterial. The final transfer matrix are obtained:

\[
[M(d)]^N = \prod_{i=1}^{N} M_i = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix},
\]

In this equation, \(m_{ij}\) \((i,j = 1,2)\) are the matrix elements of \([M(d)]^N\). The transmission and reflection coefficients of the multilayer system calculated through the following equations:

\[
t = \frac{2p_0}{(m_{11} + m_{12} p_0 + m_{21} + m_{22} p_0)},
\]

\[
r = \frac{(m_{11} + m_{12} p_0 - m_{21} + m_{22} p_0)}{(m_{11} + m_{12} p_0 + m_{21} + m_{22} p_0)};
\]

where \(p_0 = n_0 \cos \theta_0\) and \(p_s = n_s \cos \theta_s\), in which \(n_s\) is the refractive index of the free space where the wave leaves the PC with the angle \(\theta_s\). The associated transmittance \(T\) and reflectance \(R\) of the multilayer are calculated by \(T = p_0/p_s \left| t \right|^2\) and \(R = \left| r \right|^2\), respectively.

Also, the absorptance of \(A\) can be given by, \(A = 1 - T - R\). The transmittance, reflectance, and absorptance of the multilayer for TM waves can be similarly obtained through \(p_t = \mu \varepsilon \cos \theta_t\), \(p_0 = \cos \theta_t/n_0\) and \(p_s = \cos \theta_t/n_s\).

Our investigation based on two different types of DNG metamaterials. The following equations give the complex permittivity and permeability of type-I DNG layer with negative refracting index in the microwave region [17, 24],

\[
\varepsilon(f) = 1 + \frac{s^2}{0.92^2 - f^2 - if\gamma_e} + \frac{10^2}{11.5^2 - f^2 - if\gamma_e},
\]

\[
\mu(f) = 1 + \frac{3^2}{0.902^2 - f^2 - if\gamma_m}.
\]

\[\text{Figure 1}:\ A\ 1D\ MetaPC\ structure\ immersed\ in\ free\ space\ with\ a\ defect\ structure\ at\ the\ center,\ where\ layer\ A\ and\ C\ are\ the\ DNG\ metamaterial,\ layer\ B\ is\ the\ DPS\ material,\ and\ the\ incident\ angle\ is\ \theta_0.\ N\ and\ M\ are\ respectively\ the\ number\ of\ lattice\ period\ of\ the\ (AB)\ and\ (CB)\ bilayers.\]
More details concerning the various aspect of the real parts of the permittivity and permeability, $\varepsilon'$ and $\mu'$, versus frequency have discussed in our previous reports [24, 25]. The complex permittivity and the permeability will be described within the framework of the Drude model for type-II DNG layer [50] as,

\[
\varepsilon(f) = 1 - \frac{100}{f^2 - i f \gamma_e}, \quad (7)
\]

\[
\mu(f) = 1.44 - \frac{100}{f^2 - i f \gamma_m}. \quad (8)
\]

In these equations, Eqs. (5) to (8), f is the frequency given in GHz, $\gamma_e$ and $\gamma_m$ are respectively, the electric and magnetic damping frequencies (also called the electric and magnetic loss factors).

3. NUMERICAL RESULTS AND DISCUSSION

The absorption spectra of the 1D lossy defective Meta PC structures with a defect structure (PW defect) at the center is acquired by using the theoretical model depicted in the previous section. We find some anti-resonant modes (defect modes), in the region where the real parts of the permittivity and permeability of DNG layer are simultaneously negative such that the zero-$\pi$ gap will appear. Eqs. (5) and (6) are used for the permittivity and permeability of layer $A$. The DPS layer (layer $B$) considered as the vacuum layer with $n_p = 1$. Moreover, in PW defect structure, the permittivity and permeability of the DNG layer (layer $C$) is considered through Eqs. (5) and (6) for type-I DNG and Eqs (7) and (8) for type-II DNG. Also, the thickness of layers $A$, $B$, and $C$ are taken to be $d_A = 6 \text{mm}$ and $d_B = d_C = 12 \text{mm}$, respectively. The total number of the lattice period of the host structure is considered to be $N = 16$ [24]. In the analysis that follows we shall investigate the absorption property of the structure under the normal angle of incidence.

We first consider a 1D lossy MetaPC structure without any defect structure to survey the transmission, reflection, and absorption spectra. The transmission, reflection, and absorption spectra as a function of incidence frequency for $\gamma_e = \gamma_m = 2 \times 10^{-2} \text{GHz}$ illustrated in Figure 2. As it observed from the associated structure, an incident electromagnetic wave not only is transmitted and reflected but also a part of it is absorbed.

In the first part, we consider a 1D lossy MetaPC structure composed of a type-I DNG metamaterial PQW defect at the center of the host structure. Here, let us investigate the effect of the number of unit cells $M$ (which corresponds to the central defect structure) on the absorption in detail. The changes in defect modes and absorption spectra as a function of incidence frequency illustrated for various values of $M = 1, 2, 3, 4$ in Figure 3. As it is perceived, by increasing the value of $M$, the absorption slightly increases, and as we expected, the number of defect modes in the band gap increases. Furthermore, the defect modes are blue-shifted, i.e., shifted toward the upper frequency, when $M$ increased.

In Figure 4, the effect of the magnetic and electric loss factors corresponding to the type-I DNG defect layers on the rate of the absorption is surveyed when $\gamma_e = \gamma_m = 2 \times 10^{-2} \text{GHz}$. As expected, it can be seen that with a simultaneous increase in $\gamma_m$ and $\gamma_e$, the absorption increases. Moreover, according to the results mentioned in [42], the rate of transmission is dependent on both loss factors so that it will decrease as the value of $\gamma_m$ and $\gamma_e$ increases. The reflection, moreover, decreases as both loss factors increase. The further details of the changes in the absorption values corresponding to Figure 4 versus changes in the two loss factors of DNG metamaterial layer is shown in Table 1.

In the second part, we investigate the absorption spectra under different values of unit cells $M$, whereas the central defect structure contains type-II DNG layer.
Figure 5 demonstrates the changes in defect modes and absorption as a function of incidence frequency under various values of $M = 1, 2, 3,$ and $4.$ We observe, exactly like the previous part, as the number of unit cells increases, there appears an increase in the number of defect modes as well as the rate of the absorption. It is noteworthy that the number of defect modes emerging within the band gap, except for $M = 1,$ are odd numbers for different values of $M.$ Moreover, contrary to the previous part, the defect modes are red-shifted when the unit cell of the defect structure increase. Comparing with the results mentioned in the previous part in Figure 3, we clearly see the absorption of the electromagnetic wave in the MetaPC containing defect structure is enhanced when the defect structure contains type-II DNG layer, a fact that can result from the difference in the dispersion relations in $\varepsilon$ and $\mu$ for these two types of DNG metamaterial defect structures. The type-I is a Lorentz form whereas it is a Drude type for the type-II.
The role played by four different values of $\gamma_m$ and $\gamma_e$ in the rate of the absorption is examined in Figure 6, whereas the number of unit cells is assumed $M = 1$. As expected, by increasing the value of $\gamma_m$ and $\gamma_e$, the absorption increases significantly. Based on the results in Figure 6, we see that the effect of $\gamma_m$ and $\gamma_e$ on the absorption of the defect modes appearing in low frequencies is more pronounced compared with that of the other defect modes. According to the previous results [42], the effect of $\gamma_e$ on the rate of the absorption is more pronounced compared with the effect of $\gamma_m$. The further details of the changes in the absorption values (shown in Figure 6) versus changes in the transmittance values (shown in Table 2) indicate the reflectance values. In the table, the 1st and 2nd mentioned items respectively correspond to the defect mode appearing at lower and higher frequencies. As it is clearly known from the table, in the present structure, when the loss factors increase, as we expect, the absorption and transmission respectively increase and decrease, but the reflection decreases.

Table 1: The Transmittance, Reflectance, and Absorptance Values by Increasing the Magnetic and Electric Loss Factors of type-I DNG Metamaterial Defect Structure, When the Loss Factors of the Host Photonic Crystals are $\gamma_e = \gamma_m = 2 \times 10^{-2}$ GHz.

<table>
<thead>
<tr>
<th>Loss factor (GHz)</th>
<th>Absorptance</th>
<th>Transmittance</th>
<th>Reflectance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_e = \gamma_m = 0$</td>
<td>0.338</td>
<td>0.611</td>
<td>0.051</td>
</tr>
<tr>
<td>$\gamma_e = \gamma_m = 0.1 \times 10^{-2}$</td>
<td>0.359</td>
<td>0.573</td>
<td>0.068</td>
</tr>
<tr>
<td>$\gamma_e = \gamma_m = 0.5 \times 10^{-2}$</td>
<td>0.396</td>
<td>0.534</td>
<td>0.070</td>
</tr>
<tr>
<td>$\gamma_e = \gamma_m = 0.8 \times 10^{-2}$</td>
<td>0.452</td>
<td>0.457</td>
<td>0.091</td>
</tr>
<tr>
<td>$\gamma_e = \gamma_m = 1 \times 10^{-2}$</td>
<td>0.508</td>
<td>0.287</td>
<td>0.205</td>
</tr>
</tbody>
</table>

Figure 5: The absorption spectra as a function of incidence frequency for a 1D MetaPC structure containing a PQW defect with type-II DNG metamaterial layer (red solid line) and without any defect structure (blue dotted line) under various values of $M = 1$, 2, 3, and 4.
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Figure 6: The absorption spectra as a function of frequency for a 1D MetaPC structure with type-II DNG metamaterial layer PQW defect, whereas \( M = 1 \) for different values of \( \gamma_m \) and \( \gamma_e \).

Before drawing a conclusion, we compare the electromagnetic properties for the two above-mentioned structures. Based on the results in Figures 4 and 6, also in Tables 1 and 2, we find that by increasing the value of both magnetic and electric loss factors; the absorption increases, but the transmission and reflection show a contrary trend. In addition, the results show that the reflection of the electromagnetic wave in a 1D MetaPC containing a PQW defect with type-II DNG metamaterial layer is extremely higher compared with that of the other structure. It happens mainly because of the difference in the dispersion relations of \( \varepsilon \) and \( \mu \) of DNG metamaterial defect.

4. CONCLUSION

In this paper, we have theoretically investigated the absorption property of two different types of DNG metamaterial defect layers of the PQW defect in 1D MetaPCs defective structures. Our numerical results show that by increasing the defect structure’s unit cell, the rate of absorption slightly increases, and the AT modes increase in number and show blue shifts when the type-II DNG defect layer consider. The results show that with changing the types of DNG metamaterial in the PQW defect, as the number of unit cells increases, the number of AT modes and the absorption increase. But on the contrary, the appearing AT modes appear to be red-shifted. Accordingly, the results of this study lead to the conclusion that the number of AT modes strongly depends on the number of PQW’s unit cell regardless of the types of DNG metamaterial defect layer. Moreover, the AT modes are blue-shifted in the type-I DNG defect and red-shifted as a function of the type-II DNG defect. Furthermore, the results demonstrate that the absorption depends on the value of loss factors and type of DNG metamaterial. Detailed analysis of the absorption property and AT modes in such defective MetaPCs with PQW defect is scientifically significant in that it will be helpful in designing new types of tunable narrowband and multichannel filters at microwave frequency.

Table 2: The Transmittance, Reflectance, and Absorptance Values by Increasing the Magnetic and Electric Loss Factors of Type-II DNG Metamaterial Defect Structure, When Loss Factors of Host Photonic Crystals is \( \gamma_e = \gamma_m = 2 \times 10^{-2} \) GHz

<table>
<thead>
<tr>
<th>Loss Factor (GHz)</th>
<th>Absorptance</th>
<th>Transmittance</th>
<th>Reflectance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1(^{\text{st}})</td>
<td>2(^{\text{nd}})</td>
<td>1(^{\text{st}})</td>
</tr>
<tr>
<td>( \gamma_e = \gamma_m = 0 )</td>
<td>0.365</td>
<td>0.242</td>
<td>0.338</td>
</tr>
<tr>
<td>( \gamma_e = \gamma_m = 0.1 \times 10^{-2} )</td>
<td>0.526</td>
<td>0.402</td>
<td>0.276</td>
</tr>
<tr>
<td>( \gamma_e = \gamma_m = 0.5 \times 10^{-2} )</td>
<td>0.738</td>
<td>0.522</td>
<td>0.209</td>
</tr>
<tr>
<td>( \gamma_e = \gamma_m = 0.5 \times 10^{-2} )</td>
<td>0.813</td>
<td>0.623</td>
<td>0.167</td>
</tr>
<tr>
<td>( \gamma_e = \gamma_m = 1 \times 10^{-2} )</td>
<td>0.858</td>
<td>0.751</td>
<td>0.124</td>
</tr>
</tbody>
</table>
REFERENCES


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