Interacting Two Fluid Viscous Dark Energy Cosmological Models in Bianchi Type II Universe

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Abstract: In this paper, we present a class of solutions of Barber's second self creation field equations describing two fluid models of the universe in locally rotationally symmetric Bianchi type II universe filled with the barotropic fluid and bulk viscous dark energy fluid. Exact solutions of the field equations are obtained for non-interacting and interacting two fluid models. The Physical behavior of the model has been discussed.

Keywords: Bianchi type II space time, Barotropic fluid, Viscous Dark energy.

1. INTRODUCTION

Barber [1] has been produced two continuous self-creation theories by modifying the Brans- Dicke theory and Einstein's general theory of relativity. The first theory was not accepted because it's violets the equivalence principle. In the second theory, the gravitational coupling of the Einstein's field equation is allowed to be a variable scalar on the space-time manifold. The scalar field does not directly gravitate but simply divides the matter tensor, acting as a reciprocal gravitational constant. It is postulated that this scalar field couples with the trace of energy-momentum tensor. Bianchi type II space time plays an important role in current modern cosmology for simplification and description of the large scale behaviuor of the actual universe. Katore *et al*. [2] have studied cosmological models with bulk viscosity in Barber's second self creation theory. Adhav [3], Rao *et al.* [4] have investigated cosmological models for Bianchi type II space time.

Recently, cosmological observations such as Type Ia Supernovae (SNe Ia) [5, 6], the Wilkinson Microwave Anisotropy Probe (WMAP) [7], the Sloan Digital Sky Survey (SDSS) [8] indicate that our universe is in accelerated expansion. The astrophysical observations also indicate that cosmic medium is not a perfect fluid [9] and the viscosity effect could be concerned in the evolution of the universe [10]. The mysterious term dark energy is assumed to be responsible for acceleration of the universe. In the standard cosmological model, there are several dark energy models which can be distinguished by their EoS ($p_{de} = w\rho_{de}$) during the evolution of the universe. The candidate for dark energy which can be simply explained the observed cosmic acceleration is cosmological constant. Other candidates are quintessence, phantom and quintom etc.

Coley *et al.* [11] have investigated two fluid Bianchi type VI0 models. Pant *et al.* [12] have presented a class of solutions of Einstein's field equation describing two-fluid models of the universe in a locally rotationally symmetric Bianchi type II space-time. Setare *et al.* [13] have studied the tachyon cosmology in non-interacting and interacting cases in non-flat FRW universe. Adhav *et al.* [14, 15] studied anisotropic, homogeneous two fluid cosmological models in a Bianchi type I and III space-time. Amirhashchi *et al.* [16, 17], Pradhan *et al.* [18, 19] and Saha *et al.* [20] have studied the two fluid scenario for dark energy in FRW universe in different context. Singh *et al.* [21] have studied an interacting two fluid scenario for dark energy in a Bianchi type I cosmological model. Amirhashchi *et al.* [22] have studied the evolution of the dark energy parameter within the scope of a spatially non-flat and isotropic Freidman-Robertson-Walker (FRW) model filled with barotropic fluid and bulk viscous stresses. Reddy *et al.* [23] have studied the two fluid scenarios for dark energy model. Amirhashchi *et al.* [24] have investigated the role of two fluids either minimally or directly coupled in the evolution of the dark energy parameter.

The above considerations motivate us to investigate the evolution of the dark energy parameter within the framework of a Bianchi type II cosmological model filled with two fluids (barotropic fluid and bulk viscous stresses). The out line of the paper is as follows: In section 2, the metric and the field equations are described. Section 3 deals with the non-interacting two fluid model and section 4 covers the interacting two fluids model. Finally, in the conclusion is summarized in the last section 5.

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2. THE METRIC AND FIELD EQUATIONS

We consider a spatially homogeneous Bianchi type II metric [4] of the form

$$ds^2 = -dt^2 + B^2 dx^2 + A^2 dy^2 + (A^2 + y^2 B^2) dz^2 + 2yB^2 dx dz \quad (1)$$

where B, A are metric potential with respect to cosmic time 't' only. The co-ordinates of space-time are $(x^1, x^2, x^3, x^4) = (x, y, z, t)$ respectively.

The Einstein's field equations in Barber's second self –creation theory [1] are

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi\phi^{-1} T_{ij} \quad (2)$$

$$\Box\phi = \phi^{;k}_{;k} = \frac{8\pi\eta T}{3}, \quad (3)$$

where $\Box\phi = \phi^{;k}_{;k}$ is the invariant d'Alembertian and *T* is the trace of the energy-momentum tensor that describes all non-gravitational and non-scalar field matter and energy. Here η is a coupling constant to be determined from experiments. The measurement of the deflection of light restricts the value of the coupling to $|\eta| \le 0.1$. In the limit $\eta \to 0$ this theory approaches to the Einstein's general theory in every respect.

The energy momentum- tensor for a two fluid source is given [22] by

$$T_i^j = (\rho + \bar{p}) u_i^j + \bar{p} g_i^j \quad (4)$$

where T_i^j is the two fluid energy momentum tensor due to bulk viscous dark energy and barotropic fluids

$$\bar{p} = p - \xi u^j_{;i}, \quad u^i u_i = -1 \quad (5)$$

where ρ is the energy density; p is the pressure, ξ the bulk- viscous co-efficient, u^i the four velocity vector of the distribution and the semi-colon denotes covariant differentiation.

Hence equation (5) leads to

$$\bar{p} = p - \xi\left(\frac{B_4}{B} + 2\frac{A_4}{A}\right) \quad (6)$$

$$\bar{p} = p - 3\xi H, \quad (7)$$

where *H* is Hubble constant defined by

$$H = \frac{1}{3}\left(\frac{B_4}{B} + 2\frac{A_4}{A}\right). \quad (8)$$

This condition has also been used by Banerjee *et al.* [25] for deriving a viscous fluid cosmological model.

In a co-moving system of co-ordinates the field equation (2), using equation (4) and (5) for the metric (1) can be written as

$$-2\frac{A_{44}}{A} - \frac{A_4^2}{A^2} + \frac{3}{4}\frac{B^2}{A^4} = -8\pi\phi^{-1}\bar{p} \quad (9)$$

$$-\frac{A_{44}}{A} - \frac{A_4 B_4}{AB} - \frac{B_{44}}{B} - \frac{1}{4}\frac{B^2}{A^4} = -8\pi\phi^{-1}\bar{p} \quad (10)$$

$$-2\frac{A_4 B_4}{AB} - \frac{A_4^2}{A^2} + \frac{1}{4}\frac{B^2}{A^4} = 8\pi\phi^{-1}\rho \quad (11)$$

In space-time (1), the Bianchi identity for the bulk viscous fluid distribution $G^{;j}_{ij} = 0$ leads to $T^{;j}_{ij} = 0$ which yields

$$\rho u^i + (\rho + \bar{p}) u^i_{;i} = 0 \quad (12)$$

Equation (12) further reduces to

$$\rho_4 + \left(\frac{B_4}{B} + 2\frac{A_4}{A}\right)(\bar{p} + \rho) = 0, \quad (13)$$

where $\bar{p} = p_m + \bar{p}_D$ and $\rho = \rho_m + \rho_D$. (14)

Here p_m and ρ_m are the pressure and energy density of barotropic fluid and $\bar{p}_D$ and ρ_D are pressure and energy density of dark fluid respectively.

The equation of state (EoS) of the barotropic fluid w_m and dark fluid w_D are (Singh *et al.* [21]; Amirhashchi *et al.* [22]) given by

$$w_m = \frac{p_m}{\rho_m} \quad (15)$$

$$w_D = \frac{\bar{p}_D}{\rho_D} \text{ respectively.} \quad (16)$$

From equations (9) and (10), we get

$$\frac{B_{44}}{B}-\frac{A_{44}}{A}+\frac{A_4B_4}{AB}-\frac{A_4^2}{A^2}=-\frac{B^2}{A^4} \quad . \tag{17}$$

In order to solve the field equations, we use a physical condition the expansion scalar θ is proportional to the shear scalar σ which (Sharif *et al.* [26], Katore *et al.* [27]), leads to

$$B=A^n \tag{18}$$

where $n(>0)$ is a constant.

$$2A_{44}+2(n+1)\frac{A_4^2}{A}=\frac{-2}{n-1}A^{2n-3}. \tag{19}$$

Put $A_4=C(A)$. Therefore $A_{44}=CC'$

$$2CC'+2(n+1)\frac{C^2}{A}=\frac{-2}{n-1}A^{2n-3} \tag{20}$$

From equation (20), we obtain

$$C^2=\frac{1}{A^{2(n+1)}}\left[k^2-\frac{1}{2n(n-1)}A^{4n}\right]. \tag{21}$$

In particular $n=2$, we have

$$C=\frac{1}{A^3}\left[k^2-\frac{1}{4}A^{4\times 2}\right]^{\frac{1}{2}}. \tag{22}$$

Equation (22) gives us

$$A=(2k)^{\frac{1}{4}}[\sin(2t)]^{\frac{1}{4}} \tag{23}$$

$$B=(2k)^{\frac{1}{2}}[\sin(2t)]^{\frac{1}{2}} \tag{24}$$

In the following sections we deal with two cases, (i) the non- interacting two-fluid model and (ii) the interacting two fluid model

3. NON-INTERACTING TWO-FLUID MODEL

In this section we assume that two-fluids do not interact with each other. Therefore, the general form of conservation equation (13) leads us to write the conservation equation for the barotropic and dark fluid separately as

$$(\rho_m)_4+\left(\frac{B_4}{B}+2\frac{A_4}{A}\right)(p_m+\rho_m)=0 \tag{25}$$

$$(\rho_D)_4+\left(\frac{B_4}{B}+2\frac{A_4}{A}\right)(p_D+\rho_D)=0 \tag{26}$$

From equation (15) and (25) we get

$$\frac{(\rho_m)_4}{\rho_m}=-\left(\frac{B_4}{B}+2\frac{A_4}{A}\right)(1+w_m) \tag{27}$$

Equation (27) leads to

$$\rho_m=\rho_0\left[BA^2\right]^{-(1+w_m)} \tag{28}$$

$$\rho_m=\rho_0\left[(2k)\sin(2t)\right]^{-(1+w_m)}, \tag{29}$$

where ρ_0 is an integrating constant. Using equation (29) in equation (15), we obtain

$$p_m=w_m\rho_0\left[(2k)\sin(2t)\right]^{-(1+w_m)}. \tag{30}$$

The power law relation between scale factor '*a*' and scalar field ϕ has already been used by Johri and Desikan [28] in the context of Robertson Walker Brans-Dicke models.

Thus the power law relation between ϕ and '*a*' *i.e.* $\phi\propto a^n$,where *n* is any integer implies that

$$\phi=ba^n, \tag{31}$$

where *b* is the constant of proportionality.

From equation (11) and (31), we get

$$\rho=\frac{b(2k)^{\frac{n}{3}}}{8\pi}\left[\sin(2t)\right]^{\frac{n}{3}}\left[\frac{-5}{4}\cot^2(2t)+\frac{1}{4}\right] \tag{32}$$

Using equation (28) and (31) in equation (14), we have

$$\rho_D=\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[\frac{-5}{4}\cot^2(2t)+\frac{1}{4}\right]-\rho_0\left[(2k)\sin(2t)\right]^{-(1+w_m)} \tag{33}$$

Making use of equations (10), (23), (24) and (31) we obtain

$$\bar{p}=\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[\frac{-5}{4}\cot^2(2t)+\frac{1}{4}\right] \tag{34}$$

Using equations (30) and (34) in equation (14), we get

$$\bar{p}_D=\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[\frac{-5}{4}\cot^2(2t)+\frac{1}{4}\right]-w_m\rho_0\left[(2k)\sin(2t)\right]^{-(1+w_m)} \tag{35}$$

We observe that the energy density of barotropic

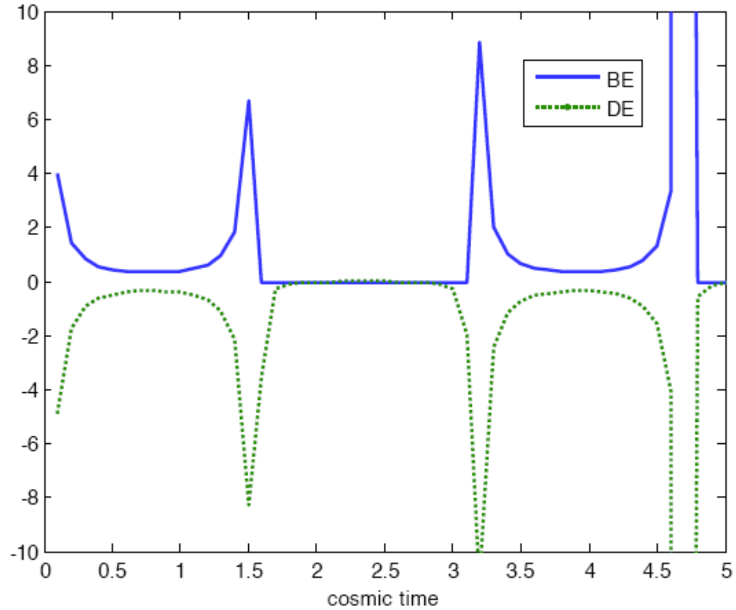

Figure 1: The plots of barotropic energy density and dark energy density versus cosmic time with $\rho_0 = b = k = n = 1, w_m = 0.5$.

fluid decreases and at a specific time the universe begins to oscillate forever due to sinusoidal property. It is remarkable to mention here that in this case oscillation takes place in the positive quadrant which is physically meaningful. Also energy density of dark energy oscillates in negative quadrant [29] as shown in Figure **1**.

From equation (16), (33) and (35) we found

$$w_D = \frac{\left[\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[-\frac{5}{4}\cot^2(2t)+\frac{1}{4}\right]-w_m\rho_0\left[(2k)\sin(2t)\right]^{-(1+w_m)}\right]}{\left[\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[-\frac{5}{4}\cot^2(2t)+\frac{1}{4}\right]-\rho_0\left[(2k)\sin(2t)\right]^{-(1+w_m)}\right]} \quad (36)$$

Therefore the effective EoS parameter for viscous dark energy can be written as

$$w_D^{eff} = w_D - \frac{\xi\left(\frac{B_4}{B}+2\frac{A_4}{A}\right)}{\rho_D} \quad (37)$$

Equation (37) reduces

$$w_D^{eff} = \frac{\left[\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[-\frac{5}{4}\cot^2(2t)+\frac{1}{4}\right]-w_m\rho_0\left[(2k)\sin(2t)\right]^{-(1+w_m)}\right] + \left[2\xi\cot(2t)\right]}{\left[\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[-\frac{5}{4}\cot^2(2t)+\frac{1}{4}\right]-\rho_0\left[(2k)\sin(2t)\right]^{-(1+w_m)}\right]} \quad (38)$$

It is observed that, the effective EoS parameter is in an oscillatory form as shown in Figure **2**. The effective EoS parameter of the DE oscillates in the matter region and the quintessence region $(w_D^{eff} > -1)$ for the large bulk viscosity co-efficient. It is clear that viscosity impacts the evolution of the universe. Thus our DE model is in good agreement with well established theoretical result as well as the recent observation Bennett *et al.* [3]; Perlmutter *et al.* [5]; Seljak *et al.* [8]; Pradhan *et al.* [18]; Riess *et al.* [30]; Eisenstein *et al.* [31]; Kumar *et al.* [32].

The expression for Hubble parameter

$$H = \frac{2}{3}\cot(2t) \quad (39)$$

The expressions for the matter-energy density of barotropic Ω_m is given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{3\rho_0(2k\sin(2t))^{-(1+w_m)}}{4\cot^2(2t)}. \quad (40)$$

The expressions for the dark energy density Ω_D is given by

$$\Omega_D = \frac{\rho_D}{3H^2} = \frac{3\left[\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[\frac{-5}{4}\cot^2(2t)+\frac{1}{4}\right]-\rho_0\left[(2k)\sin(2t)\right]^{-(1+w_m)}\right]}{4\cot^2(2t)} \quad (41)$$

From equations (40) and (41), we obtain

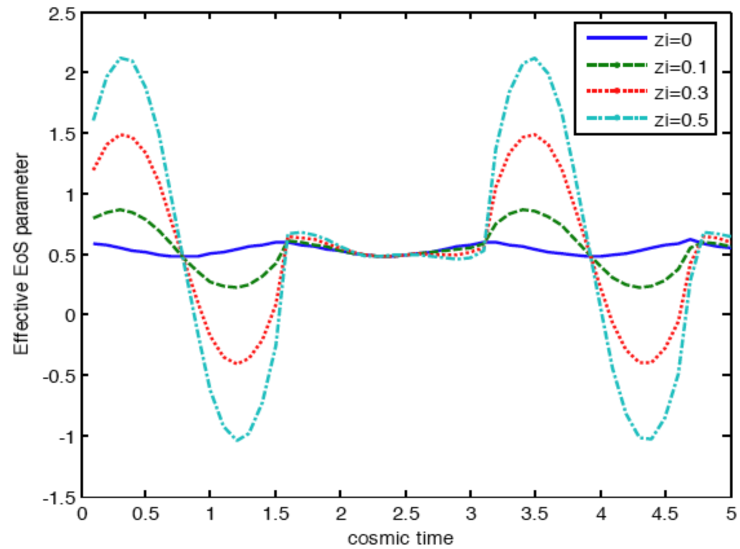

Figure 2: The plot of effective EoS parameter for DE (w_D^{eff}) versus cosmic time with $\rho_0 = b = k = n = 1, w_m = 0.5.$

$$\Omega = \frac{-15}{16} b \left[\frac{k}{4\pi} \sin(2t) \right]^{\frac{n}{3}} + \frac{3}{128} \frac{b}{\pi} \frac{\left[2k \sin(2t) \right]^{\frac{n}{3}}}{\cot^2(2t)} \tag{42}$$

4. INTERACTING TWO-FLUID MODEL

In this section, we consider the interaction between dark viscous fluid and barotropic fluid. For this purpose we can write the continuity equations for barotropic and dark viscous fluids (Singh *et al.* [21]; Eisenstein *et al.* [31]) as

$$(\rho_m)_4 + \left(\frac{B_4}{B} + 2\frac{A_4}{A} \right)(p_m + \rho_m) = Q \tag{43}$$

$$(\rho_D)_4 + \left(\frac{B_4}{B} + 2\frac{A_4}{A} \right)(p_D + \rho_D) = -Q \,, \tag{44}$$

where the quantity Q expresses the interaction between the dark components. We consider $Q>0$ which ensures that the second law of thermodynamics is fulfilled [33]. The continuity equation (43) and (44) imply that the interaction term (Q) should be

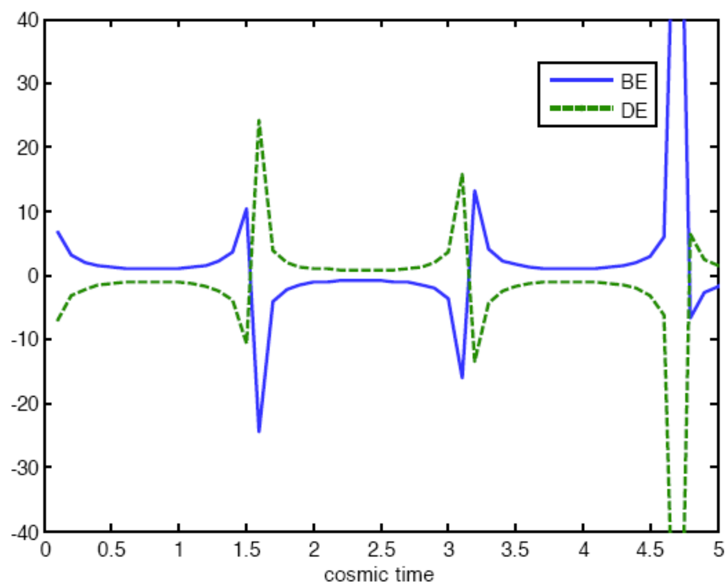

Figure 3: The plots of barotropic energy and dark energy density versus cosmic time t with $\rho_0 = b = k = 1, w_m = 0.5, n = 6, \sigma = 0.3$.

proportional to inverse of time *i.e.* $Q \propto \frac{1}{t}$.

Following to Amendola *et al.* [34] and Gou *et al* [35] we consider

$$Q = 3H\sigma\rho_m, \tag{45}$$

where σ is a coupling constant. Using equation (45) in equation (43), we obtain

$$\rho_m = \rho_0\left[\sin(2t)\right]^{-(1+w_m-\sigma)} \tag{46}$$

Using equations (32) and (46) in equation (14), we get

$$\rho_D = \frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[\frac{-5}{4}\cot^2(2t)+\frac{1}{4}\right]-\rho_0\left[\sin(2t)\right]^{-(1+w_m-\sigma)} \tag{47}$$

Using equations (16) and (32) in equation (14), we obtain

$$\bar{p}_D = \frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[\frac{-5}{4}\cot^2(2t)+\frac{1}{4}\right]+w_m\rho_0\left[\sin(2t)\right]^{-(1+w_m-\sigma)} \tag{48}$$

The behavior of the barotropic energy and dark energy density in terms of cosmic time *t* is as shown in Figure **3**. It is observed that their nature is oscillatory. Using equations (47) and (48) in equation (16). We can find the EoS of dark energy as

$$w_D = \frac{\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[\frac{5}{4}\cot^2(2t)-\frac{1}{4}\right]+w_m\rho_0\left[\sin(2t)\right]^{-(1+w_m-\sigma)}}{\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[\frac{5}{4}\cot^2(2t)-\frac{1}{4}\right]+\rho_0\left[\sin(2t)\right]^{-(1+w_m-\sigma)}} \tag{49}$$

Again we can write the effective EoS parameter of viscous dark energy as

$$w_D^{eff} = \frac{\left[\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[\frac{5}{4}\cot^2(2t)-\frac{1}{4}\right]+w_m\rho_0\left[\sin(2t)\right]^{-(1+w_m-\sigma)}\right]+2\xi\cot(2t)}{\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[\frac{5}{4}\cot^2(2t)-\frac{1}{4}\right]+\rho_0\left[\sin(2t)\right]^{-(1+w_m-\sigma)}}$$

(50)

The behavior of the effective EoS parameter for dark energy in terms of cosmic time is depicted in Figure **4**. We fix the coupling constant $\sigma = 0.3$ and bulk viscosity coefficient ξ (zi) = 0, 0.1, 0.3 and 0.5 etc. The nature of the evolution of (w_D^{eff}) is oscillatory which is physically meaningful [29]. It should be note that the variation of effective EoS parameter is in positive quadrant which shows that the universe is matter dominated.

In Figure **5**, we fix the viscosity parameter $\xi = 0.5$ and coupling constant σ (sigma) = 0.1, 0.5, 1 and 1.5 etc. It is observed that, the (w_D^{eff}) is in a oscillatory form [29]. The effective EoS parameter of the DE begins in the matter region and crosses the phantom divide (PDL) or cosmological constant $(w_D = -1)$ region and then passes over into the phantom $(w_D < -1)$ region for the large value. Thus

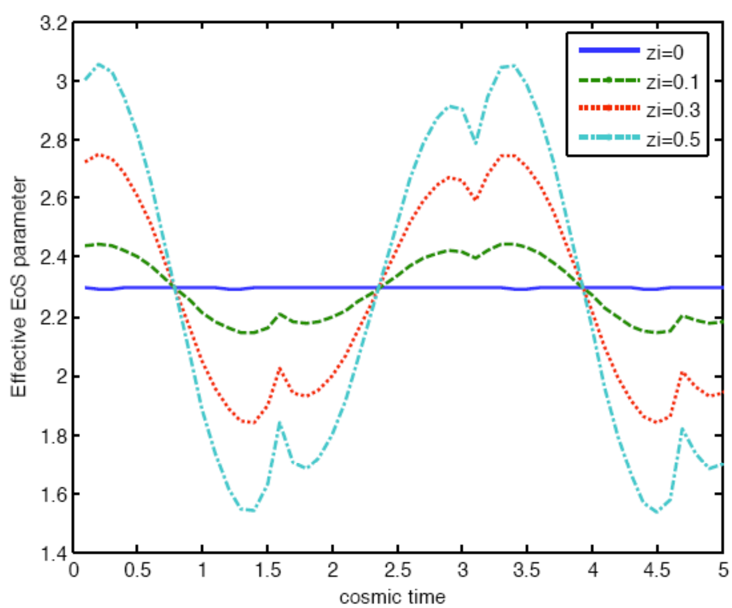

Figure 4: The plot of EoS parameter for DE (w_D^{eff}) versus cosmic time with $\rho_0 = b = k = 1, w_m = 0.5, \sigma = 0.3, n = 6$.

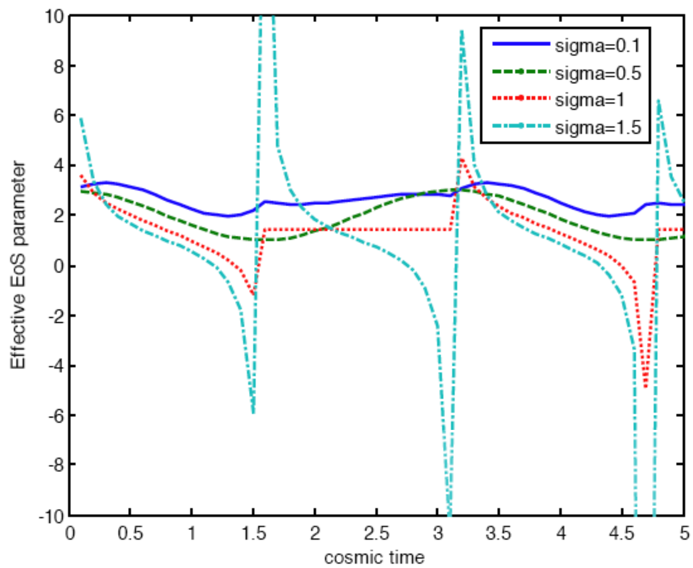

Figure 5: The plot of EoS parameter for DE (w_D^{eff}) versus cosmic time (t) with $\rho_0 = b = k = 1, w_m = \xi = 0.5, n = 6$.

σ brings impact on the evolution of the universe. Our DE model is in good agreement with well established theoretical result as well as the recent observations Bennett *et al.* [3]; Perlmutter *et al* [5]; Seljak *et al.* [8]; Pradhan *et al.* [18]; Riess *et al.* [30]; Eisenstein *et al.* [31]; Kumar *et al.* [32].

The expressions for the matter energy density Ω_m is given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{\rho_0 (\sin(2t))^{-(1+w_m-\sigma)}}{4\cot^2(2t)} \tag{51}$$

The expressions for the dark energy density Ω_D is given by

$$\Omega_D = \frac{\rho_D}{3H^2} = \frac{-3\left[\frac{b}{8\pi}\left[(2k)\sin(2t)\right]^{\frac{n}{3}}\left[\frac{5}{4}\cot^2(2t) - \frac{1}{4}\right] + \rho_0\left[\sin(2t)\right]^{-(1+w_m-\sigma)}\right]}{4\cot^2(2t)} \tag{52}$$

Adding equation (51) and (52), we get total energy density

$$\Omega = \frac{3b}{128\pi}\left[2k\sin(2t)\right]^{\frac{n}{3}}\left\{-5 + \frac{1}{\cot^2(2t)}\right\} \tag{53}$$

5. CONCLUSIONS

We have studied the evolution of dark energy parameters within the framework of the universe in locally rotationally symmetric Bianchi type II model filled with barotropic fluid and bulk viscous dark energy .The exact solutions of the Barber's second self creations field equations have been obtained by assuming two different non-interacting and interacting two fluid models. The behavior of these models has been analyzed and it is observed that:

(i) In non-interacting and interacting models, the energy density of barotropic fluid oscillates in positive quadrant and energy density of dark energy oscillates in negative, which result resembles to Pradhan *et al*. [29].

(ii) In the non-interacting model, the effective EoS parameter for DE is in an oscillatory form. It oscillates in the matter region and the quintessence region for large value of the bulk viscosity co-efficient. Thus, it is clear that bulk viscosity impacts the evolution of the universe.

(iii) In the interacting model, the effective EoS parameter is in positive quadrant which shows that the universe is matter dominated whereas effective EoS parameter oscillates between matter region and phantom region which is consistent with recent observation.

(iv) It is interesting to note that in the absence of scalar field, our results resembles to the investigated results of Amirhashchi *et al.* [22]. This study will throw some light on the structure formation of the universe, which has astrophysical significance.

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REFERENCES

[1] Barber GA, Gen. Relativ. Grav. 1982; 14: 227.

[2] Katore SD., Rane R.S., Wankhade KS., Int. J. Theor Phys. 2010; 49: 187-193.

[3] Adhav KS. Electronic Journal of Theoretical Physics 2012; 9, No. 26: 239-250.

[4] Rao VUM, Sireesha KVS, Neelima D. ISRN Astronomy and Astrophysics, 2013; Vol. 2013: Article I D 924834, 11 pages.

[5] Perlmutter S, Gabi S, Goldhaber G, Goobar A, Groom DE. Hook I.M. *et al,* ApJ, 1997; 483: 565.

[6] Riess AG, Filippenko AV, Challis P, Clocchiatti A, Diercks A, Garnavich P.M. *et al.* (Supernova Search Team Collaboration) ApJ. 1998; 116, 1009.

[7] Bennett CL, Halpern M, Hinshaw G, Jarosik N, kogut A, Limon M. *et al.* ApJS, 2003; 148: 1.

[8] Seljak, U, Makarov, A., McDonald, P., Anderson, S.F., Bahcall, N.A., Brinkmann, B.J. *et al.,* Phys. Rev. D. 2005; 71: 103515.

[9] Jeffe TR, Banday AJ, Eriksen HK, Gorski KM, Hansen, FK, *et al.* Astrophysical J 2005; 629: L1-L4.

[10] Cataldo M, Cruz N, Lepe S. Phys Lett B 2005; 619: 5.

[11] Coley AA, Dunn K. Astrophys J 1990; 348: 26.

[12] Pant DN, Sanjay Oli. Astrophysics and Spce sci 2002; 281: 623.

[13] Setare MR, Sadeghi J, Amani RR. Phys Lett B 2009; 673: 241.

[14] Adhav KS, Dawande MV, Borikar SM. Bulg J Phys 2011; 38: 371-379.

[15] Adhav KS, Borikar SM, Desale MS, Raut RB. Electronic Journal of Theoretical Physics 2011; 8, No. 25: 319-326.

[16] Amirhashchi H, Pradhan A, Saha B. Chinese physics Lett 2011; 28: 039801.

[17] Amirhashchi H, Pradhan A, Zainuddin H. Int J of Theoretical Physics 2011; 50: 3529.

[18] Pradhan A, Amirhashchi H. Modern Phys Lett A 2011; 30: 2261.

[19] Pradhan A, Amirhashchi H, Saha B. Astrophysics and space sci 2011; 333: 343.

[20] Saha B, Amirhashchi H, Pradhan A. Astrophysics and space sci 2012; 342: 257-267, DoI: 10.1007/s10509-012-1155-x.

[21] Singh T, Chaubey R. Research in Astronomy and Astrophysics 2012; 12: 473.

[22] Amirhashchi H, Pradhan A, Zainuddin H. Research in Astron. Astrophys 2013; Vol.13, No. 2: 129-138 ,arXiv, 1210.4637v3 [astr-ph CO] (24 Oct 2012).

[23] Reddy DRK, Anitha S, Umadevi S. Astrophysics Space sci DoI: 10.1007/s/0509-014-1778-1.

[24] Amirhashchi H, Chouhan DS, Pradhan A. EJTP 2014, 11, No. 30, 109-126.

[25] Banerjee A, Duttachoudary SB, Sanyal AK. Gen Relat and Gravit 1986; 18: 461.

[26] Sharif M., Zubair M. Astrophysics Space Sci, 2010; 330: 399-405.

[27] Katore SD, Shaikh AY. Bulg J phys 2012; 39: 241-247.

[28] Johri VB, Desikan K. Gen Relativ Grav 1994; 26:1217.

[29] Pradhan A, Otarod S. Astro Phys sci 2007; 311: 413-421.

[30] Riess, AG, Strolger, LG, Tonry J, Casertano S, Ferguson, HC, Mobasher, B, *et al.* Ap J 2004; 607: 665.

[31] Eisenstein DJ, Zehavi I, Hogg DW, Scoccimarro R, Blanton, MR, Nichol RC, *et al.* Astrophs J 2005; 633: 560.

[32] Kumar S, Yadav AK. Modern Physics Letters A 2011; 26: 647

[33] Zhao GB, Xia JQ, Li H, Tao C, Virey JM, Zhu ZH, *et al.,* Physics Letters B. 2007; 648:8

[34] Amendola L, Campos GC, Rosenfeld R, *et al.* Phys. Rev. D. 2007; 75: 083506

[35] Guo ZK, Ohta N, Tsujikawa S. Phys Rev D 2007; 76: 023508.

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