

# Undamped, Damped and Forced Motion, Physical Simulation with the Geogebra Tool

Reynaldo Campos Saravia<sup>1</sup>, José Antonio Manco Chávez<sup>2\*</sup>, Nicolas Silver Manco Arroyo<sup>3</sup>, Jorge Antonio Crisóstomo Olivares<sup>4</sup>, Jeannina Del Alva Huaman Carhuatocto<sup>5</sup>, Luz Petronila Blas Montenegro<sup>6</sup>, Mary Rosaura Díaz Hinostraza<sup>7</sup>, Nancy Barazorda Puga<sup>8</sup>

<sup>1,2,5</sup>*Universidad Autónoma del Perú, Lima, Perú.*

<sup>3</sup>*Universidad Peruana de Ciencias Aplicadas, Lima, Perú.*

<sup>4</sup>*Universidad Tecnológica del Perú, Lima, Perú.*

<sup>6</sup>*Universidad Católica Sedes Sapientiae, Lima, Perú.*

<sup>7</sup>*Universidad Nacional Tecnológico de Lima Sur, Lima, Perú.*

<sup>8</sup>*Universidad Privada del Norte, Lima, Perú.*

**Abstracts:** Currently due to the pandemic situation that has affected the world community without distinction of languages, customs and creeds, technology was incorporated massively to achieve the continuity of progress in research, and development of culture and academic education, through this progress. As an introduction we can mention that this research is achieved by the need to do physics using other tools easy to manipulate with basic programming knowledge, likewise in the repositories there are many other studies that applied computationally the development of simulations of different physical phenomena. The method to use is the GeoGebra tools applying the basic syntax that is obtained by entering the equations in the input bar of the GeoGebra software already preset, being these equations calculated many times by students using differential equations, which obtains the solution equation dependent on mass and time, in this way the simulation, undamped, damped and forced motion are achieved, with the aim to show in this research that physics can be done using tools for academic use, due to this software is not exactly a research tool, but it can simulate phenomena that have already been shown in experimental and computational laboratories. The results obtained are very interesting since they present an originality in its simulation in an objective way showing dynamically the oscillation and the appearance of the functions that correspond to it in the development of the movement of the spring mass system.

**Keywords:** Damped motion, Hooke's law, Simple pendulum, Undamped motion, Forced motion.

## 1. INTRODUCTION

Within the field of engineering, technologies (Rodríguez-Cardoso et al Farías et al, 2020), pure science and medicine, we can find a large number of existing applications in the construction of equipment that help to measure pulses and waves with and harmonic behavior (Torres Rodríguez,2020), damped and forced taken to be reduced by the Fourier analysis in engineering practice to describe the behavior of these signals on a screen; within this field of study the research will be developed using differential equations ( Espinoza,2020) as well as derivatives and integrals; for better study analysis and among other natural phenomena in man's life that occur with the participation of this phenomena within a disturbance (Gazzola & Llanos,2020), also its application in electronic engineering, telecommunication and remote sensing.

In the field of pure and applied sciences such as engineering, the M.A.S, damped and forced motion are often used as we can find in books that are world class in research such as books on physics in oscillations and waves, Phyton scripting for computational science and physics, numerical methods in this field with Python and among others such as the book of numerical methods in engineering with Python, it is mentioned that these materials publish their ways of conducting research on this phenomenon (Gazzola Bascougnat & Llanos,2020), but so far it is no found a work in GeoGebra on these topics in research.

The aim of this research is to build these physical models with a tool that is given little importance and yet few educators use this tool to model or perform any simulation for their classes (Bascougnat & Llanos,2020). In this study, it is found that teachers use this tool at the training level in user mode, there are few researchers who bring this new tool of teaching into the classroom, generating in students their own concerns in research, which is innate to the search for knowledge in human beings, and thus making them manage to create new knowledge. At the same time, tabulations were made in the GeoGebra spreadsheet to show different results that are subject to the investigative

variables of the M.A.S and damping; this study will allow students and teachers to generate knowledge and links the learning of equations of the physical models to take them to the programming using GeoGebra.

The applications of damped motion are used to measure the movement of the earth (Lema Sanango,2021), the rotation of satellites and earth on its own axis that is inclined by an angle of less than 10 degrees sexagesimal, likewise, the damped motion is used to measure the pressure exerted by forced movements in solid bodies such as bridges that experience pressures capable of generating a destruction of them, these types of research are considered for high applicability in materials and their behaviors, due to these movements are also used to measure distances or depth in civil engineering and among others scientific and technological disciplines.

A mechanical oscillator model bound to Hooke's law is not close to reality, because they are ideal models that are not subjected to resistance forces and those that are in turn, making important changes in the oscillations that occur at different times.

Likewise, researchers such as (Lighezzolo et al,2019), mentioned that it is necessary the parameterization of physical equations within technological tools that facilitate the construction of the technology in the simulation according to the needs of the researcher, this implies a cognitive development with future researchers working in industries as, refer (Li et al,2022), in the same way to achieve mastery or empowerment of technology based on simulations making the construction of high performance generators is achieved.

The aim in this study is to perform the modeling of damped oscillations in objects since these are subjected to resistances and that after a certain period they stop. This modeling is done with the intention of representing real situations. The general problem found in this study, despite the great amount of information to develop new knowledge applied with technology is still a privilege for few people, the objective is to show how practical is doing research using GeoGebra software with the topic "Damped and forced motion, physical modeling solved with GeoGebra tool". Likewise, the specific problem to develop in this study is to show the importance of the GeoGebra use in the "Damped and forced motion, physical modeling solved with GeoGebra tool".

GeoGebra contributes with knowledge to the "Damped and forced motion, physical modeling solved with the GeoGebra tool", in engineering careers. In the same way, it is raised the general objective in the development of the investigation of simple, damped and forced movements. To obtain the simulation of the phenomena, damped and forced movements with the GeoGebra tool, for better understanding and studying in the field of technological innovations.

This research is justified due to the need to obtain new ways of doing physics in university classrooms. The purpose of this study is to know how to use GeoGebra tools through the appropriate operations with its syntax and the help shown by this software to enter the basic commands in the input bar and thus be able to execute the physical modeling of undamped motion, damped and forced motions. Physical simulations are performed by applying methodologies analytically on physical phenomena and mathematical calculations that occur on the damped and forced motion, likewise this technology integrates knowledge of all specialties in the field of engineering and training in university education as referred by (Manco-Chavez et al,2020), who shows that the integration of ICT is necessary for the development of new approaches in vocational training.

In the field of applied research such as engineering and pure sciences, it allows the development and innovation of new types of knowledge with the use of technological tools (Ocaña-Fernández et al, 2020) and the understanding of theoretical models' structures in physics.

To understand this, we must first know the term simple harmonic motion, which is any periodic motion of a system obeyed by Hooke's law (Montañés Muñoz et al,2020), and after a period of time, the cycle of travel of a particle is repeated again; being constant and maintained for an infinite time without decreasing its amplitude; this occurs because there is a restoring force that is proportionally direct to the displacement regarding to its equilibrium point

and that allows to develop elastic potential energy accumulated by the spring at the time of the circular motion, which exerts the particle or mass subject to the spring.

This process and conditions for a simple harmonic motion to occur within the research (León & González,2020), are the following: in all circular motion where the speed is constant, it experiences a radial or central force in charge of making it keep performing the circumferential motion, this force change the direction of the mass subject to the spring, as described by (Giancoli,2008).

The system considered in GeoGebra at the present time does not consider the friction force in the studies of some research, but in practice, the systems are subject to some of these forces mentioned; managing to generate that the amplitude of the oscillations is decreasing over time due to the restoring force that presents all elastic material such as the spring.

To describe these simulations in the physical system using GeoGebra, it will consider the friction forces for different cases applied (Giancoli,2008), in addition will be considered idealized or friction forces with different coefficients. The damping is considered as the decrease of the amplitude generated by the dissipative forces, and the damped oscillation is considered as the movement that occurs (Sears et al, 2009).

Damped oscillators are subject to energy losses due to the friction that occurs at the time of their oscillations since the amplitude has an exponential decay behavior as time passed (Sears Z et al, 2009) and (Giancoli,2008); this is understood as a progressive decrease in the amplitude of the oscillation until it stops completely.

Considering that the damped motion has a sinusoidal shape that decreases as times goes by, the following equations are proposed to define plot a damped oscillation respecting to specific initial variables that will be mentioned, according to Newton's second law.

To understand this, we must ask ourselves what is damped motion, first it must be understood the term of simple harmonic motion, which is any periodic motion of a system obeyed by Hooke's law and after a certain time, the cycle of the particle travel is repeated again; being constant and maintained for an infinite time without decreasing its amplitude; this occurs because there is a restoring force  $F$  which is proportionally direct to the displacement  $X$  with respect to it equilibrium point. See Image 1.

$$\text{Hooke's law: } = -kx, k > 0, k: \text{ coefficient of elasticity} \quad (1)$$

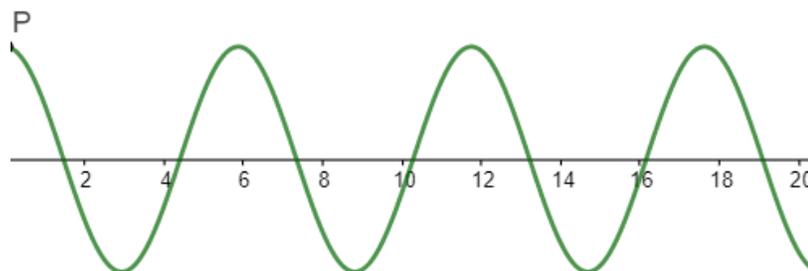
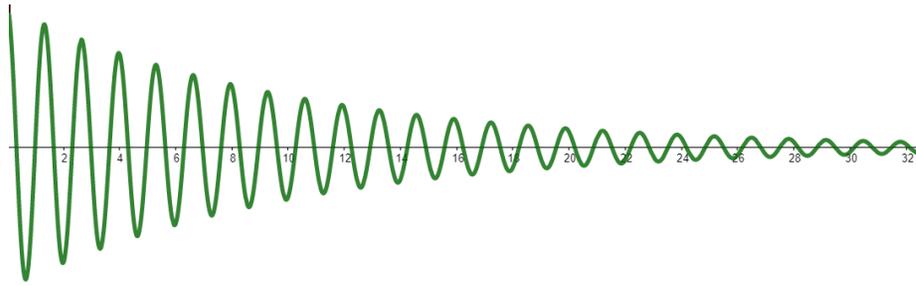


Figure 1. Simple Harmonic Motion (SHM).

According to (Garzón,2022), the system used at the present does not consider the friction but in practice, systems are subject to this; generating that the amplitude of the oscillations decreases. The damping is considered as the

decrease of the amplitude generated by the dissipative forces; and the damped oscillation is considered as the movement that occurs. See Image 2.



**Figure 2.** Damped Motion.

We will add to this type of motion the friction, which is that value that depends on the speed of the degree of freedom of  $x$ . This force is responsible for the change from a simple harmonic motion to a damped motion.

$$\text{Friction: } F_R = -bv \tag{2}$$

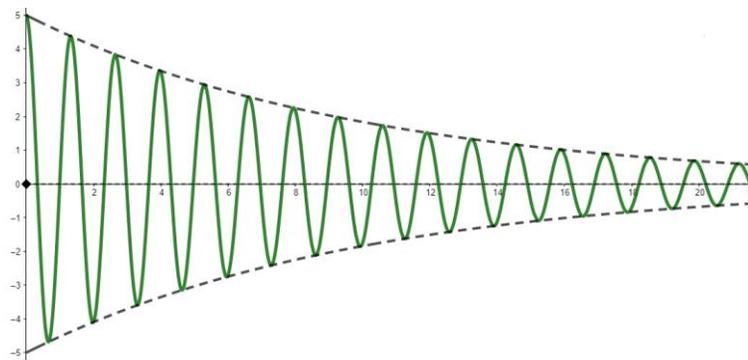
$$\text{Net force: } F_T = kx - bv \tag{3}$$

b: Damping constant

According to (De la Torre et al, 2015), it should be mentioned that the amplitude is the main characteristic in the oscillation of the spring mass system, but especially of the damped oscillation because the amplitude of this decreases exponentially by the time. Therefore, the energy of the oscillator also decreases. In the presented phase space, the motion describes a spiral converging to the origin.

The damped oscillators are subject to energy losses due to the friction that occurs at the time of their oscillations since the amplitude has an exponential decay behavior as time passes; this is understood as a progressive decrease in the amplitude of the oscillation until it stops completely. See Image 3.

$$\text{Amplitude Equation: } A_0 e^{-\frac{b}{2m}t} \tag{4}$$



**Figure 3.** Damped Motion Subject to Energy Losses.

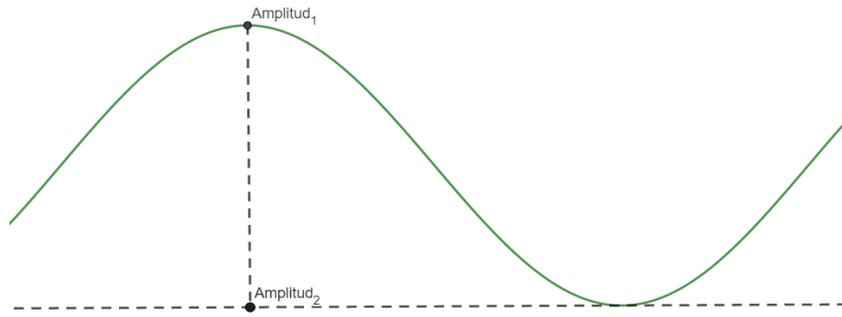


Figure 4. Amplitude of Undamped Motion (AUM).

Likewise, the period is the time it takes for the mass to complete one cycle (perform one complete oscillation). It is represented by the letter T.

### Frequency

While the frequency is the number of cycles per unit time performed by the damped oscillation. It is related to the period and its formula is as follows:

$$\text{Angular frequency: } f = \frac{1}{T} \tag{5}$$

$$\text{Angular frequency when there is no damping: } w_0 = \sqrt{\frac{k}{m}} \tag{6}$$

$$\text{Angular frequency when damping is present: } w' = \sqrt{w_0^2 - \left(\frac{b}{2m}\right)^2} \tag{7}$$

Considering that the damped motion has a cosine form that decreases as time passes, the following equations are proposed to define and plot a damped oscillation of the defined initial variables that will be mentioned.

The following image represents the variation (mismatch) of the angle regarding to the damped motion which is represented with the Greek letter  $\phi$ . See Image 5.

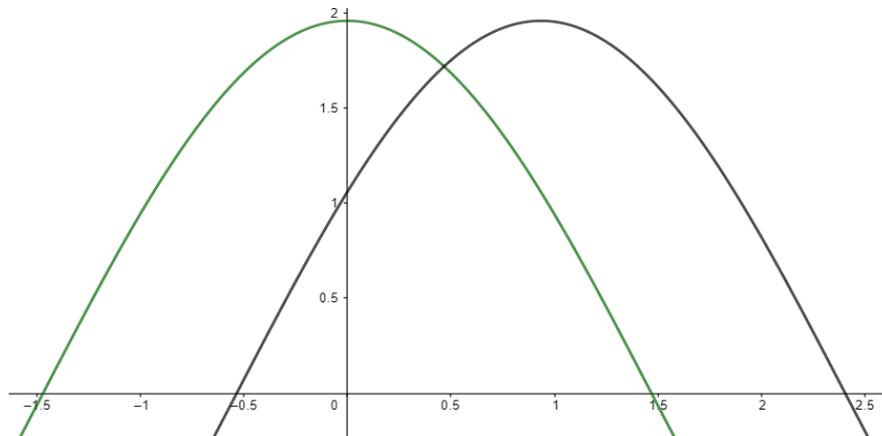


Figure 5. Angle of mismatch in the SHM.

Considering that the damped motion has a cosine form where the equation decreases as time passes, the following equations are proposed to define and simulate a damped oscillation regarding to the defines initial variables that will be mentioned.

$$\text{According to the Newton's law: } \sum F_x = ma_x \quad (8)$$

It is known that the net force is represented as follows:

$$F_T = -kx \quad (9)$$

$$\text{Then: } \sum F_x = F_T \text{ where, } ma_x = -kx \quad (10)$$

Considering the above formulas, we can obtain the speed, which is the derivative of the position at x with respect to the time, and the acceleration, which is the derivative of the speed according to time:

Applying Newton's second law, where the sum of all forces equals zero:

$$ma_x + bv + kx = 0 \quad (11)$$

Then:

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0 \quad (12)$$

By subtracting the mass m in the equation and solving this, we will consider a solution of the form  $x =$

$$e^{\gamma t}, \text{ where } \gamma = -\frac{b}{2m} \quad (13)$$

$$\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{kx}{m} = 0 \quad (14)$$

$$m(r^2 e^{\gamma t}) + b(re^{\gamma t}) + ke^{\gamma t} = 0 \quad (15)$$

$$(mr^2 + br + ke^{\gamma t})e^{\gamma t} = 0; \text{ as } e^{\gamma t} \neq 0 \quad (16)$$

$$\text{Then } mr^2 + br + ke^{\gamma t} = 0 \quad (17)$$

From equation (17) it is obtained an auxiliary equation, whose solution is:

$$\gamma = -\frac{b}{2m} \pm \frac{\sqrt{b^2 - 4mk}}{2m} \quad (18)$$

If  $b^2 - 4mk < 0$  then the auxiliary equation has complex roots, then we have the following solution. (19)

$$\gamma = -\frac{b}{2m} \pm i \frac{\sqrt{4mk - b^2}}{2m} \quad (20)$$

$$\text{From where the roots are like this: } \gamma_1 = \alpha + i\beta, \gamma_2 = \alpha - i\beta \quad (21)$$

Making comparisons according to the equation or model obtained, it is making the following equations.

$$\alpha = -\frac{b}{2m}, \beta = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = w' \quad (22)$$

Then the solution of the equation (12).

$$x(t) = c_1 e^{\alpha t} \cos \beta t + c_2 e^{\alpha t} \text{sen} \beta t \tag{23}$$

Considering the constants belonging to the amplitude, it is obtained that they are perpendicular.

$$A_0 = \sqrt{c_1^2 + c_2^2} \text{ y } \phi \text{ is a phase angle} \tag{24}$$

$$c_1 = A_0 \text{sen} \phi, c_2 = A_0 \cos \phi \tag{25}$$

Then it can be replacing the results to get the equation of motion.

$$x(t) = A_0 e^{\alpha t} \cos \beta t \cdot \text{sen} \phi + A_0 e^{\alpha t} \text{sen} \beta t \cdot \cos \phi \tag{26}$$

$$x(t) = A_0 e^{\alpha t} \text{sen}(\beta t + \phi) \tag{27}$$

Finally, replacing equation (22) in equation (27), the result is as follows:

$$x(t) = A_0 e^{-\frac{b}{2m}t} \text{sen}(\beta t + \phi) \tag{28}$$

## 2. METHODOLOGY

Due to the versatility of GeoGebra presenting this educational tool (Gutiérrez-Araujo & Castillo-Bracho, 2020), is totally different from the hard programming languages in its syntax and source codes for the elaboration of any modeling and that are even more complicated to work applied to objects that are totally dynamized, it is to say that it is possible to observe the phenomenon in movement being thus a physical modeling and that could show us results that would come to make the data of the construction through the time according to the physical phenomenon is manifested (Gañan Trejos, 2020), this software that is of easy handling in students and teachers, the physical and mathematical foundations, will be applied as the derivatives, integrals, the differential equations for the support of the methodological construction and this way to be able to verify its attributes of the language code of the software.

The methodology to be used is the programming development using the GeoGebra software that will be the tool allied to the investigation of the non-damped, damped and forced movement reproduction, these will be constructed by means of this tool that has a powerful competence directed to objects being an applicable program and of easy comprehension, it is not so simple but compared to other programs that work with consoles and it is not possible to observe the construction while not running the program, however this software will help the researcher to observe from the beginning since its construction, all the procedures and those in the line that are looking to develop.

The theoretical framework will be used and it will have the aim to perform the necessary operations to achieve the undamped, damped and forced motion as if the data had been obtained from an experimental investigation with the difference that it will be recreated with the program and it will be computer modeling that is operated by the equations obtained by solving theoretically and their sine, cosine and time-dependent energy loss approximations.

The techniques to be used are the object directed programming system, through the dynamic screen of GeoGebra.

In the procedure will be used the equations of motion, constant force and variables forces to describe the movement of this type of physical behavior that occurs daily on these behaviors applied even in technology and industry or engineering.

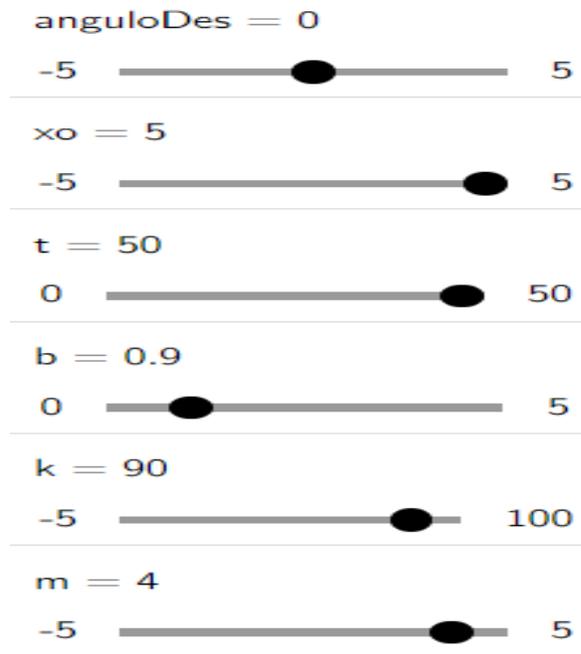
Finally, it is a study that will be developed from a computer or laptop that allows installing the GeoGebra program or software, where it will be possible to perform the operationalization of the physical and mathematical models that allow dynamizing the movement already mentioned in the title of the research project.

### 3. RESULTS

Being mentioned in the previous section the essential fundamentals for understanding damped motion, it will be proceeded to describe the modeling process in the GeoGebra tool.

Step 1- Defining the initial values in the GeoGebra slider, to make each one interactive.

First, it is defined the variables: m,k,b,t,x\_0 (Initial amplitude), a deface, entering the data in the input bar (B.E).



**Figure 6.** Sliders Created by Defects, when Entering Equations in the GeoGebra E.B.

We define equations (6) and (7) in the input bar to obtain graphically the behavior of the equations of damped motion.



**Figure 7.** Results of Entering Equations (6) and (7) in the E.B.

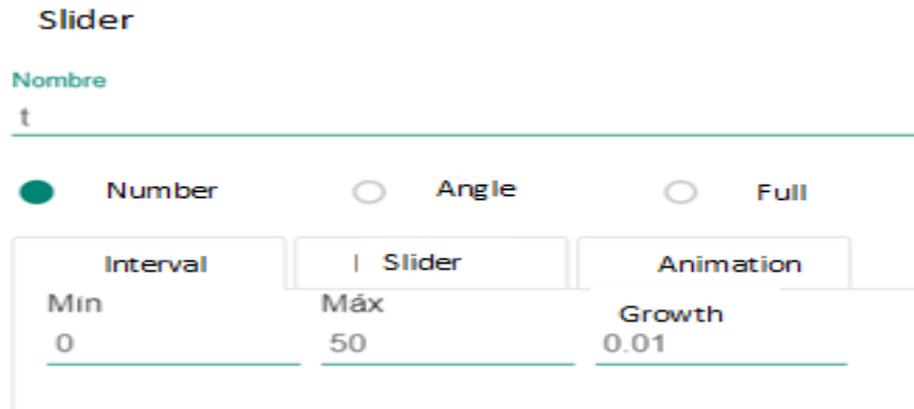


Figure 8. Setting the data to be entered in the E-B- for the creation of the sliders.

Step 2 – Drawing the graph of the damped motion.

To obtain the sine function of the damped motion it is entered the following equation in GeoGebra:

$$f(t) = x_0 e^{-\frac{b}{2m}t} \text{sen}(w't + \phi) \quad (15)$$

Result.

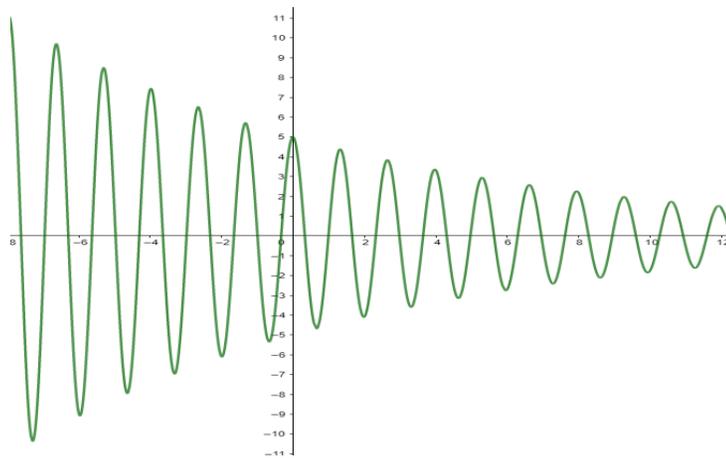
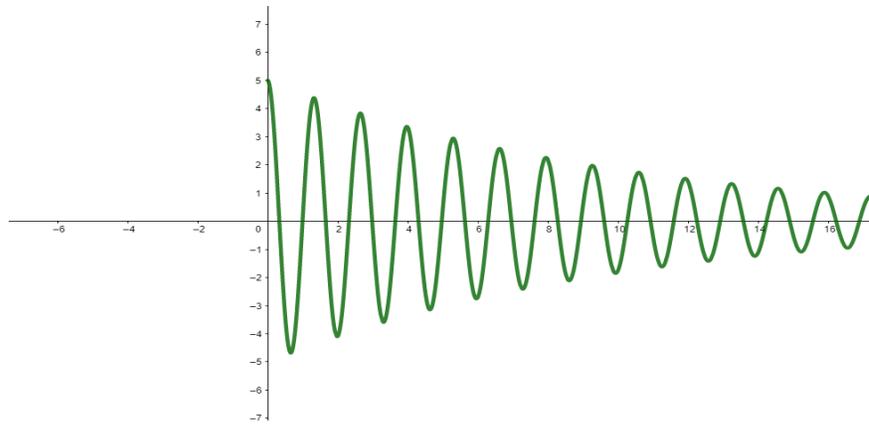


Figure 9. Graph of Equation (15).

Seeing that the generated graph includes negative values on the x-axis, it is applied a restriction to the domain in GeoGebra so that only positive values can be considered; the applied formula is new function and is defined as follows.

$$p(t) = Si(0 \leq t \leq \infty, f(t) \text{ unci}) \quad (16)$$



**Figure 10.** Graph of Equation 15, Bounded on the Positive Axis.

Likewise, it is continued with the construction in this new bounded equation, to carry out the investigation, for this it is entered the known data previously established by the equations.

Now, it is drawn the positive graph of the amplitude (y-axis), for it in GeoGebra will be entered the following function:

$$g(t) = x_0 e^{-\frac{b}{2m}t} \tag{17}$$

And then it will be drawing the negative graph of the amplitude (y-axis) through the following function.

$$h(t) = -x_0 e^{-\frac{b}{2m}t} \tag{18}$$

Noting that the generated amplitude plot includes negative values on the  $x$  axis, it is applied a restriction to the domain in GeoGebra so that only positive values can be considered; the applied formula is a new function which is defined as follows:

For positive amplitude on the y-axis:

$$q(t) = If(0 \leq t \leq \infty, g(t)unci) \tag{19}$$

For negative amplitude on the y-axis:

$$r(t) = If(0 \leq t \leq \infty, h(t)unci) \tag{20}$$

We obtain as a result.

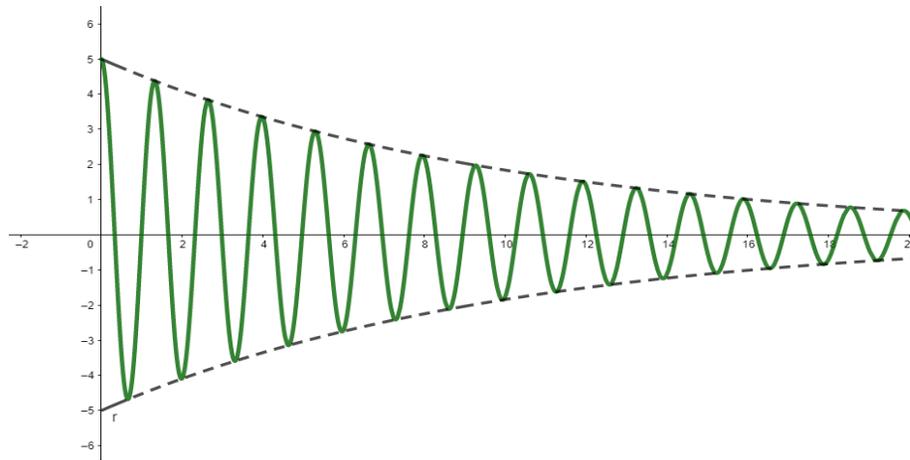


Figure 11. Result of Equations (19) and (20).

### Step 3 – Modeling mass/spring motion

It is defined a new variable in GeoGebra which will be called  $x_1$  that will store the value of the x displacement (previously known as y-axis). To do this the variable  $x_1$  will be assigned the following formula:

$$x_1 = x_0 e^{-\frac{b}{2m}t} \text{sen}(w't + \phi) \quad (21)$$

It is drawing a point in GeoGebra called Point Motion by going to the dynamic sheet, as shown in the image, this will allow the dynamic elaboration.

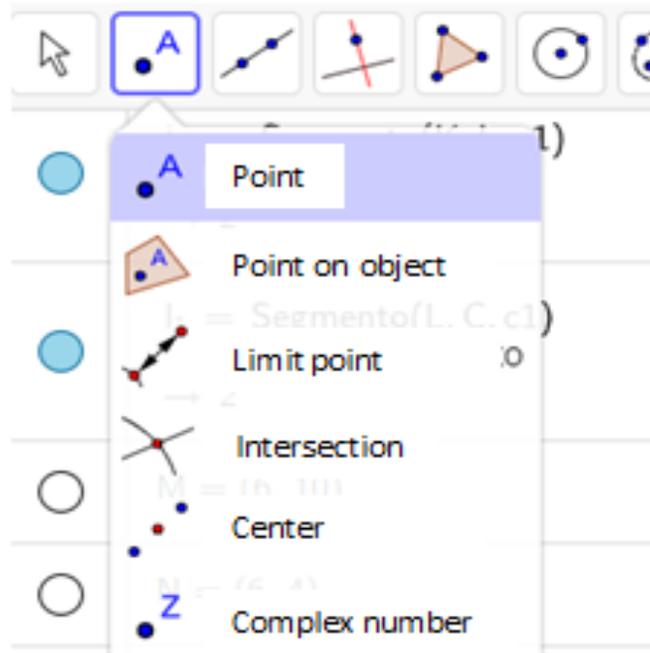


Figure11. Elaboration of the Point That Will Help to Link the Union of a Spring on a Mass.

It is assigned the following values to the Motion Point components:

$$\text{MotionPoint} = (t; x_1) \quad (22)$$

To animate the plotted MotionPoint, we must right click on the t (time) slider that we added in GeoGebra and then click on the animation option.

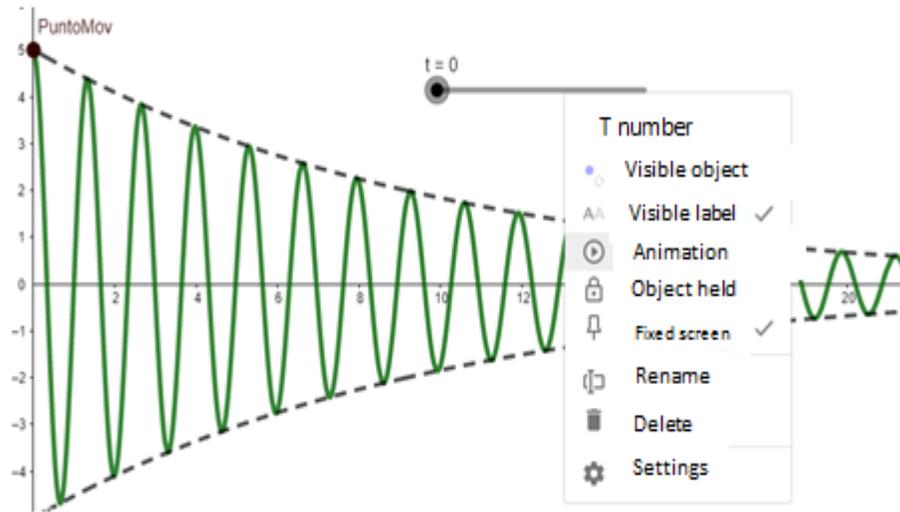


Figure 12. Animating the Time t, of the Spring Mass Motion.

Now, it will be proceeded to draw a spring with a mass (object), to associate it with the moving particle mentioned above. This is done with the intention of giving a more realistic picture of the damped motion that objects produce when they have a mass, an elasticity and damping constant.

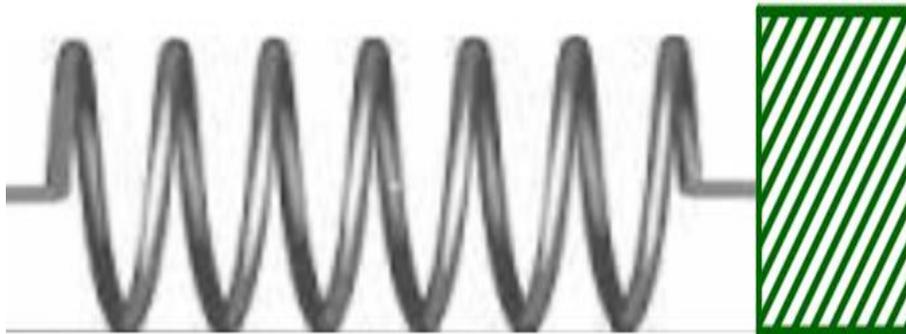


Figure 13. Linked Spring Mass System in GeoGebra.

Steps to follow to perform the spring-mass linkage:

We export the spring and mass image in the dynamic sheet of GeoGebra.

Place the lower left corner of the spring image in the coordinates (0;6) and it will have the letter B assigned to it.

Place the lower right corner of the spring image at the coordinates  $(5+x_1; 6)$  and it will have the letter C assigned to it.

Observing the results on the screen, the lower right coordinate contains in its first component the following sum:  $5+x_1$ . This is done with the aim that the spring is elastic and varies (stretches or compresses) according to the changing values of the variable  $x_1$ .

Change the height of the spring image. To do this, it is needed to go to the image settings and assign the values (0, 8), see image 14.

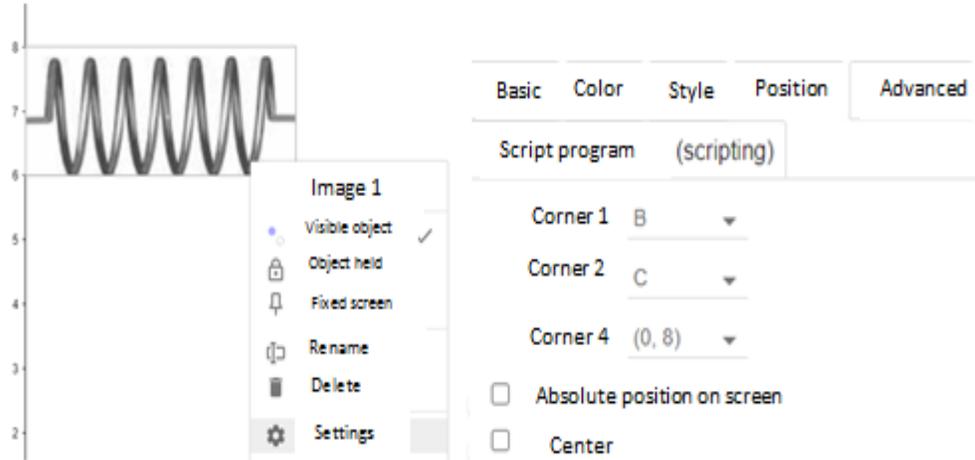


Figure 14. Configuring the Spring Image for Physical Modeling.

Now, it is exported the mass to link it with the spring, which is the green block; to do this we go to the polygon option. This will ask you to assign the coordinates of the points needed to plot the polygon, the points will be: J,K,L,C.

Assign the values to the coordinates:

Point J (upper left vertex):  $(5+x_1; 8)$

Point K (upper right vertex):  $(7+x_1; 8)$

Point L (lower right vertex):  $(7+x_1, 6)$

Point C: Point previously assigned in the image of the spring in the lower right corner and that will help for the drawn mass to move synchronously with the spring.

The result will be a dynamic square; it is said to be dynamic due to  $x_1$  is the variable that stores the value of the sine function that changes as time passes. See image 15.

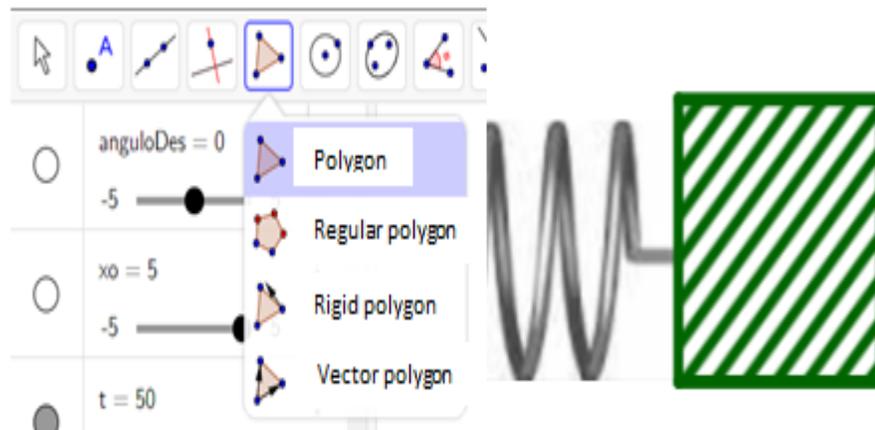
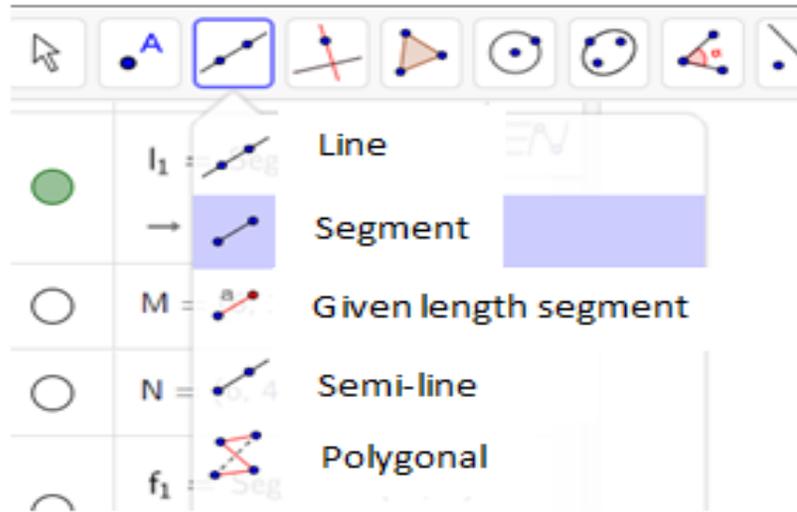


Figure 15. Mass Spring Linkpage.

Now, we will draw a segment to simulate the floor, which will be responsible for generating the friction force; to do this, we go to the Segment option.



**Figure 16.** Creation of the Segment that will Simulate the Floor where the Spring Mass System will be Placed.

The segment will have eta I assigned to it and will be defined as follows:  $I=(F,G)$ .

The values of F and G are as follows:

$$F= \text{Point}(\text{AxisY}) \quad (23)$$

$$G=(18, 6) \quad (24)$$

At this point we can animate the t (time) slider and you can see that the spring moves according to the graph of the sine function:

Step 4 – Modeling simple pendulum with damped motion.

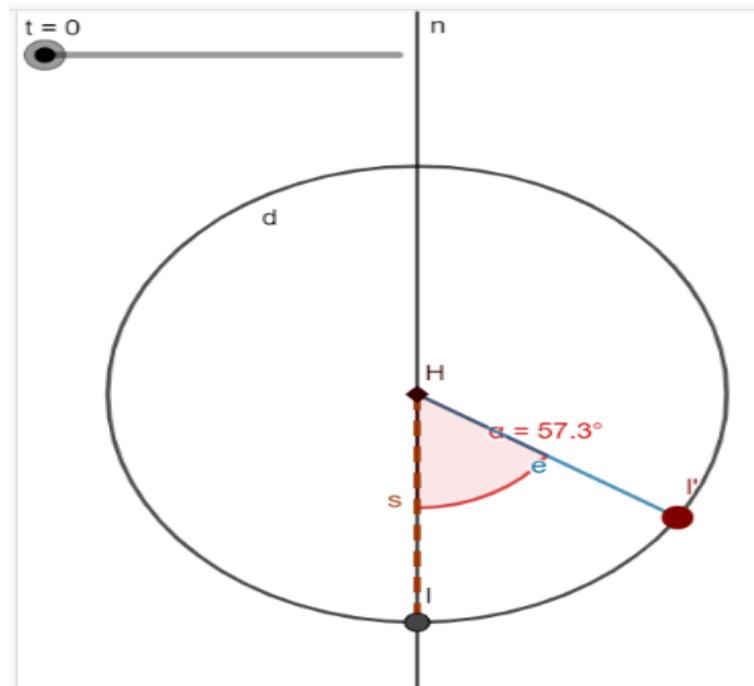


Figure 17. Behavior of the Simple Pendulum.

Now we will model a simple pendulum associated with the damped oscillation. Image 17.

Since we know GeoGebra; in the following lines we will focus more on statements and commands that GeoGebra support and that allow us to model elements in a faster and simplified way.

We located a point, which we will call H; for this case we will place it at the coordinate (7,7).

We draw a line parallel to the Y-axis at a distance from the point  $H$ , for which we type the following command:  $n: \text{Line}(H, EjeY)$ .

Add circle by inserting the command  $d: \text{Circle}(H, x_0)$ .

Generate a point of intersection between the circle  $d$  and the parallel line  $n$  by typing the following command:  $I = \text{Intersect}(d, n, 1)$ .

We draw a segment between point  $H$  and point  $I$ ; where we type the following command  $s = \text{Segment}(H, I)$ .

We declare a variable named  $c$  to store the value of the variable MotionPoint in the y-axis; we do this with the intention of associating the value of the variable  $c$  to the angle of the pendulum.

We state a variable which we will call angle; this will serve us to generate the angle of oscillation with respect to the damped movement; we will assign the following formula:  $\frac{c}{x_0}$ ; where  $c$  is the value of the variable MotionPoint in the y-axis and  $x_0$  is the initial amplitude. We apply this formula so that the angle of the pendulum is analogous to the damped oscillations of the mass/spring system we designed earlier.

We generate a rotation point, which we will call  $I'$  and will be defined by the following sentence:  $I' = \text{Rota}(I, \text{angle}, H)$ ; this will be with the intention of generating a point that models the oscillatory motion of the point  $I$  that we generated previously at the intersection of the circumference  $d$  and the parallel line  $n$ .

We draw a segment between point H and point I'; for that we must type the command  $e=\text{Segment}(H, I')$ .

We generate an angle  $\alpha$  between point I and point I'; and it will be defined by the following command:  $\alpha = \text{Angle}(I, H, I')$ .

In this way the simple pendulum would already be associated with the spring-mass system, and we could already see the modeling of an oscillatory motion of a simple pendulum.

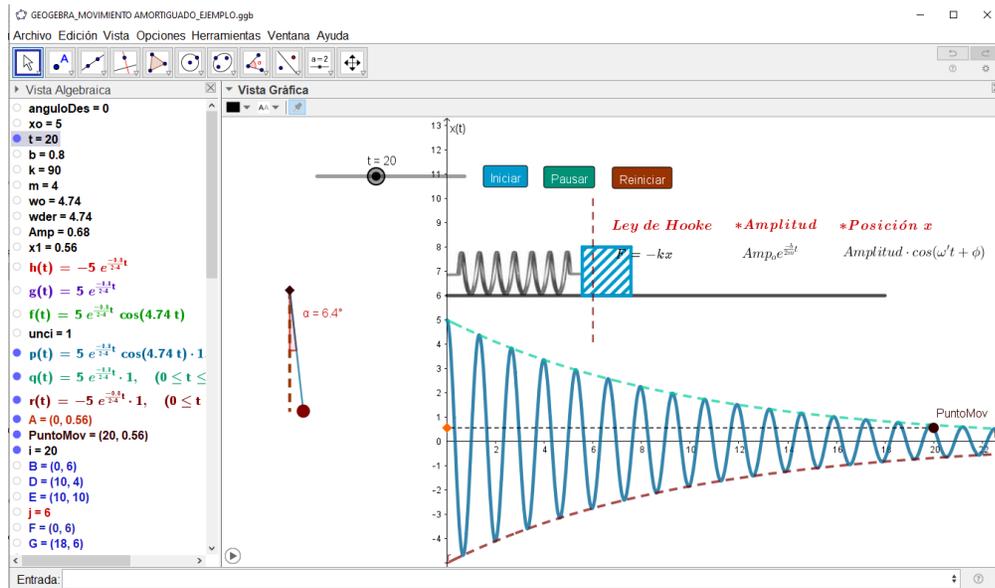


Figure 18. Simulating the Spring-Mass System.

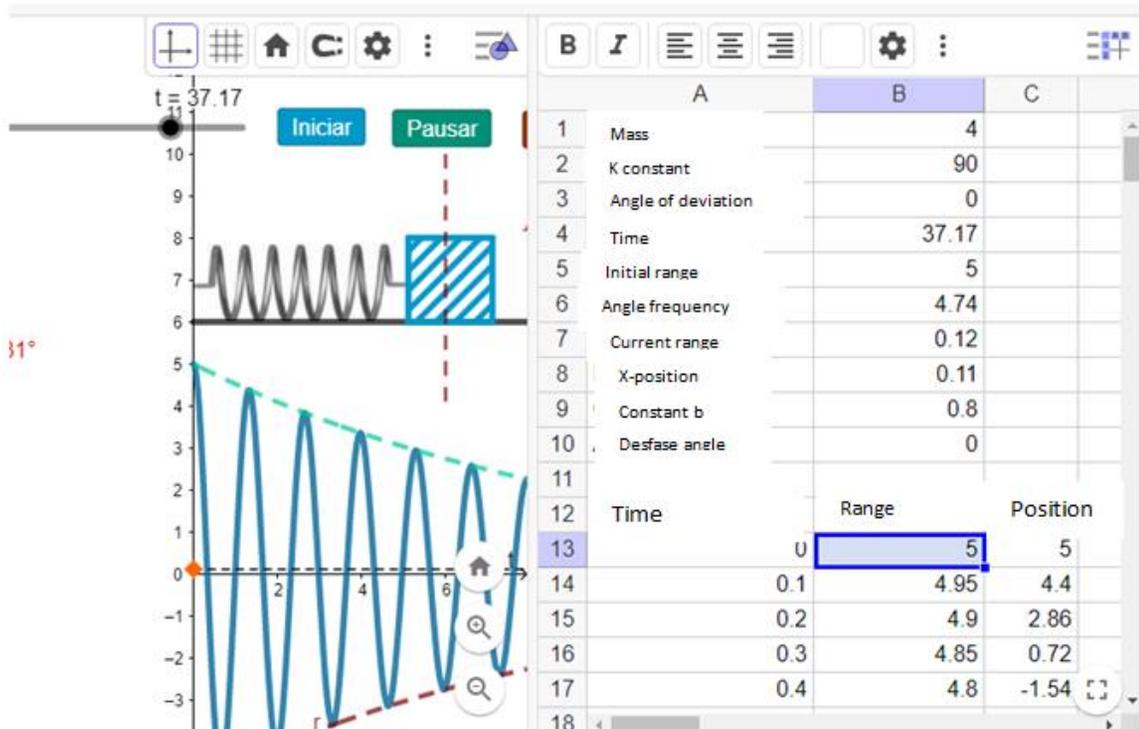


Figure 19. Simulating the Spring-Mass System.

## 4. CONCLUSIONS

According to what has been worked, it is concluded that it is possible to perform modeling and simulations of real physics situation in GeoGebra; this allows to have a clearer image of the theoretical part of the damped motion, since it allows to observe in real time the displacement of a spring or pendulum until the moment of its stop. To perform the modeling of the damped motion, it is essential to have a clear understanding of the fundamentals of simple harmonic motion, since it is under its principles that the modeling can be performed. GeoGebra is the appropriate and ideal tool for physical and mathematical modeling because it provides many tools and options. It allows to enhance the educational, research and scientific fields. Finally, this research contributes to the knowledge gap by showing the interactivity achieved by simulating a physical phenomenon using this GeoGebra tool.

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