# Temperature Fluid Flow of Particles with The Effect of Magnetic Field in A Cylindrical Polar Coordinates of Incompressible Dusty Fluid

N. Jagannadham<sup>1</sup>, B. K. Rath<sup>2</sup>, D. K. Dash<sup>3</sup>.

<sup>1</sup>Research scholar, Department of Mathematics, GIET University, Gunupur -765022

<sup>2</sup>Department of Mathematics, GIET University, Gunupur -765022.

<sup>3</sup>Head of the Department, Mathematics, CCET Bhilai, Chhattisgarh, India; E-mail: <u>njagannadham@giet.edu</u>, <u>biplabkr@giet.edu</u>, <u>drdillipdash70@gmail.com</u>.

**Abstracts:** The effects of temperature of dusty fluid flow in a weak magnetic field are investigated in this paper. The motion of dusty fluid is affected by a homogeneous magnetic field that runs in the same direction as the fluid. Parameters include longitudinal velocity, Hartmann number; dust particle interactions, stock resistance, Reynolds number, and magnetic Reynolds number fully characterize the topic that is being studied. The obtained partial differential equations are translated into an ode system using appropriate similarity transformations. To solve these equations numerically we use, the Laplace Transform technique. The result shows the temperature of fluid particle reduced significantly in the magnetic field.

Keywords: Magnetic Field, Boundary Layer, Volume Fraction, Incompressible Flow.

# Nomenclature

- f dimensionless stress function
- m magnetic parameter
- pr prandtl number
- b0 magnetic field intensity
- cf local skin friction coefficient
- re reynolds number
- t fluid temperature
- u fluid velocity along x-axis
- u0 reference velocity
- v fluid velocity along y-axis

# 1. INTRODUCTION

In addition, no one has tried to investigate consequences of physical characterizing parameters on induced magnetic profile, such as the magnetic Reynolds number, despite the fact that many researchers have observed the fluid flow from a variety of angles without focusing on the impact of the fluid flow and the generated magnetic field or on the skin friction.

The present study fills this knowledge gap by investigating the impact of an induced magnetic field on the temperature flow of incompressible dusty fluid in a tube. Dusty fluid boundary layer issues have been studied for a

long time. A number of authors, including Sakiadies (1961), Girresha (2012), and Mudassar (2017), have investigated both pulsating and the constant motion of incompressible fluids. The presence of a magnetic field has made the study of stable dusty fluid flow under fluid flow a topic in the area of applied mathematics. Several scientists have been able to analyze the Profiles of temperature, pressure, heat transmission, and dust fluid flow velocity as a function of many physical characterizing characteristics like the magnetic parameter, the parameters of the fluid particles, and the parameters of the dust particles.

This article examines the topic of boundary layer fluid flow from the perspective of a number of researchers, and it discusses a selection of their observations in order to determine the extent to which those observations and the present study diverge. This study aims to shed light on these researchers' work by introducing the magnetic induction equation to the equations describing the steady incompressible flow of dust particles through a boundary layer. This is a vital area of research that has attracted the attention of many modern mathematicians.

Using both mathematical and numerical approaches, Prandtl (1904) and Sakiadies (1961) investigated boundary layer flow and constant velocity travel, and Prandtl subsequently developed an expression for a boundary layer in two dimensions and axisymmetric flow.

He looked into how factors like pressure, suction, the Prandtl number, and the Eckert number affected the flow of fluid, among other physical parameters. This allowed him to discover that the fluid phase velocity (U) did not change significantly, while the particles phase velocity (Up) changed as the fluid particle interaction parameters increased. In addition, the temperature gradient and surface heat transfer were both enhanced for fluids with a higher Prandtl number due to the thinner thermal boundary layer. When the suction parameter was increased, he saw a drop in the fluid velocity (U), which he associated with a considerable rise in the dust particle phase velocity (Up) (fo). After reviewing the aforementioned research, Mudassar J. et al. (2017) investigated the solution of boundary flow of dusty fluid flow with an applied magnetic field. By defining his issue in terms of the fluid-particle interaction parameter ( $\beta$ ), the magnetic field parameter (M), and the dust-particle mass concentration parameter ( $\gamma$ ), he found that a rise in the magnetic parameter increased Lorentz force, causing a drop in fluid velocity.

We spoke about the incompressible fluid's heat in cylindrical polar coordinates and how a magnetic field affects it. Think about how the jet's speed and temperature varies somewhat from the steam all around it. To achieve this linearization of the governing differential equations, the perturbation approach has been used [1, 2]. The velocity of the dusty fluid is described, and the differential equations are solved using the Laplace transformation, in our previous works [8, 12]. Particle heat profiles under perturbation have been used to highlight numerical issues. This demonstrates a significant diminishment in the heat particle perturbation magnitude.

# 2. MATHEMATICAL FORMULATION

The axi-symmetric boundary layer flow regulating differential equation is represented in cylindrical polar coordinates as

Fluid-phase heat equation

$$(1-\phi)\rho C_{p}\left(u\frac{\partial T}{\partial z}+v\frac{\partial T}{\partial r}\right) = K\left(\frac{\partial^{2}T}{\partial r^{2}}+\frac{1}{r}\frac{\partial T}{\partial r}\right) + \rho_{p}C_{s}\frac{\left(T_{p}-T\right)}{\tau_{t}}$$
(1)

Heat Equation in Particle phase

$$\rho_{p} C_{p} \left( u_{p} \frac{\partial T_{p}}{\partial z} + v_{p} \frac{\partial T_{p}}{\partial r} \right) = -\rho_{p} C_{s} \frac{\left( T_{p} - T \right)}{\tau_{T}}$$
(2)

In order to study the boundary layer flow, the dimensionless variables are provided.

Are 
$$\overline{z} = \frac{z}{\lambda}$$
,  $\overline{r} = \frac{r}{(\tau_{\rm m}\upsilon)^{\frac{1}{2}}}$ ,  $\overline{u} = \frac{u}{U}$ ,  $\overline{v} = v\left(\frac{\tau_{\rm m}}{\upsilon}\right)^{\frac{1}{2}}$ ,  $\overline{u}_{\rm p} = \frac{u_{\rm p}}{U}$ ,  $\overline{v}_{\rm p} = v_{\rm p}\left(\frac{\tau_{\rm m}}{\upsilon}\right)^{\frac{1}{2}}$ ,  $\alpha = \frac{\rho_{\rm p_0}}{\rho} = \text{const}$   
 $\overline{\rho}_{\rm p} = \frac{\rho_{\rm p}}{\rho_{\rm p_0}}$ ,  $\overline{T} = \frac{T}{T_0}$ ,  $\overline{T}_{\rm p} = \frac{T_{\rm p}}{T_0}$ ,  $\lambda = \tau_{\rm m}U$ ,  $\tau_{\rm m} = \frac{2}{3}\frac{C_{\rm p}}{C_{\rm s}}\frac{1}{p_{\rm r}}\tau_{\rm T}$ ,  $p_{\rm r} = \frac{\mu C_{\rm p}}{K}$ .

We may now assume, the pressure at the exit is the same as the pressure in the surrounding stream. The temperature of the jet is just slightly higher than the nearby stream's soundscape. The viscosity coefficient and thermal conductivity K are assumed to be fixed parameters. Afterwards you'll be able to put pen to paper.

$$u = u_0 + u_1, v = v_1, u_p = u_{p_0} + u_{p_1}, v_p = v_{p_1}, T = T_0 + T_1, T_p = T_{p_0} + T_{p_1}$$

Where the subscripts 1 denote disturbed values that are noticeably smaller than the fundamental values with subscripts'0,' i.e., the surrounding stream.  $u_0 \gg u_1$ ,  $u_{p_0} \gg u_{p_1}$ ,  $T_0 \gg T_1$ ,  $T_{p_0} \gg T_{p_1}$ . After the elimination of the constant term, the nonlinear equations (1) and (2) may be rewritten in terms of the dimensionless variable and perturbation method.

$$(1-\phi) u_0 \frac{\partial T_1}{\partial z} = \frac{1}{p_r} \left( \frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} \right) + \frac{2\alpha}{3p_r} \rho_{p_1} \left( T_{p_0} - T_0 \right)$$
(3)

$$u_{p_0} \frac{\partial T_{p_1}}{\partial z} = \frac{2}{3} \frac{1}{p_r} \left[ \left( T_0 - T_{p_0} \right) + \left( T_1 - T_{p_1} \right) \right]$$
(4)

The boundary conditions for  $u_1, v_1, u_{p_1}$  and  $v_{p_1}$  are

$$u_{p_{1}}(0, r) = \begin{cases} u_{p_{10}}, & r \leq 1 \\ 0, & r > 1 \end{cases}$$
(5)

Similarly, the boundary conditions for  $\,T_{_{1}}^{}\,,\,T_{_{p_{_{1}}}}^{}\,$  are

$$T_{1}(0, r) = \begin{cases} T_{10}, & r \leq 1 \\ 0, & r > 1 \end{cases}$$
(6)

$$\frac{\partial T_{1}}{\partial r}(z,0) = 0, T_{1}(z,\infty) = 0$$
(7)

$$T_{p_{1}}(0, r) = \begin{cases} T_{p_{10}}, & r \leq 1 \\ 0, & r > 1 \end{cases}$$
(8)

Particle density  $\rho_{p_l}$  is constrained by the following boundaries:

$$\rho_{p_{1}}(0, r) = \begin{cases} \rho_{p_{10}}, & r \leq 1 \\ 0, & r > 1 \end{cases}$$
(9)

# 3. METHODLOGY:

Using the Laplace transform method, we solved the governing linearized equation (4) by plugging in the necessary conditions from (5.1) through (9).

$$u_{p_0} \frac{\partial T_{p_1}}{\partial z} = \frac{2}{3p_r} \left[ \left( T_0 - T_{p_0} \right) + \left( T_1 - T_{p_1} \right) \right]$$

Taking Laplace Transform on both sides

$$\Rightarrow u_{p_0} L\left(\frac{\partial T_{p_1}}{\partial z}\right) = \frac{2}{3p_r} \left[L\left(T_0 - T_{p_0}\right) + L\left(T_1 - T_{p_1}\right)\right]$$

$$\Rightarrow \frac{\partial T_{p_1^*}}{\partial z} = \frac{2}{3p_r u_{p_0}} K + \frac{2}{3p_r u_{p_0}} T_1^* - \frac{2}{3p_r u_{p_0}} T_{p_1}^*, \text{ where } K = \frac{T_0 - T_{p_0}}{p}$$

$$\Rightarrow \frac{\partial T_{p_1^*}}{\partial z} + \frac{2}{3p_r u_{p_0}} T_{p_1}^* = \frac{2}{3p_r u_{p_0}} \Big( K + T_1^* \Big)$$

$$\Rightarrow \frac{\partial T_{p_1^*}}{\partial z} + CT_{p_1}^* = C\left(K + T_1^*\right)$$

Where  $C = \frac{2}{3p_r u_{p_0}}$ 

Identify the differential equation of linear first order.

Find the answer by

$$I.F = e^{\int C dz}$$

The Necessary Answer is

$$T_{p_{1}^{*}} e^{\int Cdz} = \int C(K + T_{1}^{*}) e^{\int C dz} dz$$
$$= e^{-\int Cdz} \int C(K + T_{1}^{*}) e^{\int C dz} dz$$

$$=e^{-\int Cdz}\int C\left(K+T_{1}^{*}\right)e^{cz}dz$$

$$T_{p_{1}^{*}}=e^{-cz}\left[C\left(K+T_{1}^{*}\right)\frac{e^{cz}}{c}-\int\frac{\partial T_{1}^{*}}{\partial z}e^{cz}dz\right]$$

We have

~

$$T_{1}^{*} = \left(T_{10} - \frac{2\alpha E\rho_{p10}}{3Ap^{2}}\right) \frac{j_{1}(p)}{p} e^{\frac{-Ap^{2}z}{Ap^{2}}} + \frac{2\alpha E\rho_{p10}}{3Ap^{2}} \frac{j_{1}(p)}{p}$$

Now

$$\int e^{cz} \frac{\partial T_{1}^{*}}{\partial z} dz = \int -\left(\frac{AT_{10}p^{2}}{p_{r}} - \frac{2\alpha F\rho_{p10}}{3p_{r}}\right) \frac{J_{1}(p)}{p} e^{-\frac{Ap^{2}z}{pr}} e^{cz} dz$$

$$= -\int \left(\frac{AT_{10}p^{2}}{p_{r}} - \frac{2\alpha F\rho_{p10}}{3p_{r}}\right) \frac{J_{1}(p)}{p} e^{\left(c - \frac{Ap^{2}}{p_{r}}\right)z} dz$$

$$= \left(\frac{AT_{10}p^{2}}{p_{r}} - \frac{2\alpha F\rho_{p10}}{3p_{r}}\right) \frac{J_{1}(p)}{p} \frac{e^{\left(c - \frac{Ap^{2}}{p_{r}}\right)z}}{c - \frac{Ap^{2}}{p_{r}}} + D$$

$$T_{p1}^{*}(z, p) = T_{1}^{*}(z, p) + \left(\frac{AT_{10}p^{2}}{p_{r}} - \frac{2\alpha F\rho_{p10}}{3p_{r}}\right) \frac{J_{1}(p)}{p} \frac{e^{\left(c - \frac{Ap^{2}}{p_{r}}\right)z}}{c - \frac{Ap^{2}}{p_{r}}} e^{-cz} + De^{-cz}$$

When  $z \rightarrow 0$  we have

$$T_{p_{1}}^{*}(0,p) = T_{1}^{*}(0,p) + \left(\frac{AT_{10}p^{2}}{p_{r}} - \frac{2\alpha F\rho_{p10}}{3p_{r}}\right)\frac{J_{1}(p)}{p}\frac{1}{c - \frac{Ap^{2}}{p_{r}}} + D$$

$$D = \frac{T_{p10}J_1(p)}{p} - \frac{T_{10}J_1(p)}{p} - \left(\frac{AT_{10}p^2}{p_r} - \frac{2\alpha F\rho_{p10}}{3p_r}\right)\frac{J_1(p)}{p}\frac{1}{c - \frac{Ap^2}{p_r}}$$

The above equation allows us to determine.

1026

$$\begin{split} T_{p1}^{*}(z,p) &= \left(T_{10} - \frac{2\alpha F\rho_{p10}}{3Ap^{2}}\right) \frac{j_{1}(p)}{p} e^{-\frac{Ap^{2}z}{p_{r}}} + \frac{2\alpha F\rho_{p10}}{3Ap^{2}} \frac{J_{1}(p)}{p} + \left(\frac{AT_{10}p^{2}}{p_{r}} - \frac{2\alpha F\rho_{p10}}{3p_{r}}\right) \frac{J_{1}(p)}{p} \frac{e^{-\frac{Ap^{2}z}{p_{r}}}}{c - \frac{Ap^{2}}{p_{r}}} - \frac{2\alpha F\rho_{p10}}{3p_{r}} \frac{J_{1}(p)}{p} + \frac{2\alpha F\rho_{p10}}{p_{r}} + \frac{J_{1}(p)e^{-cz}}{c - \frac{Ap^{2}}{p_{r}}} - \frac{2\alpha F\rho_{p10}}{3p_{r}} \frac{J_{1}(p)}{p_{r}} \frac{e^{-cz}}{c - \frac{Ap^{2}}{p_{r}}}}{c - \frac{Ap^{2}}{p_{r}}} - \frac{2\alpha F\rho_{p10}}{3p_{r}} \frac{J_{1}(p)}{p_{r}} \frac{e^{-cz}}{c - \frac{Ap^{2}}{p_{r}}}}{c - \frac{Ap^{2}}{p_{r}}} - \frac{2\alpha F\rho_{p10}}{3p_{r}} \frac{J_{1}(p)}{p_{r}} \frac{e^{-cz}}{c - \frac{Ap^{2}}{p_{r}}}}{c - \frac{Ap^{2}}{p_{r}}} - \frac{2\alpha F\rho_{p10}}{3p_{r}} \frac{J_{1}(p)}{p_{r}} \frac{e^{-cz}}{c - \frac{Ap^{2}}{p_{r}}}}{c - \frac{Ap^{2}}{p_{r}}} - \frac{2\alpha F\rho_{p10}}{3p_{r}} \frac{J_{1}(p)}{p_{r}} + \frac{2\alpha F\rho_{p10}}{c - \frac{Ap^{2}}{p_{r}}}}{c - \frac{Ap^{2}}{p_{r}}} - \frac{2\alpha F\rho_{p10}}{3p_{r}} \frac{J_{1}(p)}{p_{r}} + \frac{2\alpha F\rho_{p10}}{c - \frac{Ap^{2}}{p_{r}}}}{c - \frac{Ap^{2}}{p_{r}}} - \frac{2\alpha F\rho_{p10}}{3p_{r}} \frac{J_{1}(p)}{p_{r}} + \frac{2\alpha F\rho_{p10}}{(c - \frac{Ap^{2}}{p_{r}})} - \frac{2\alpha F\rho_{p10}}{(c - \frac{Ap^{2}}{p_{r}})} -$$

$$p\left(c - \frac{Ap}{p_r}\right) = \frac{P_r}{p_r} = \frac{P_r}$$

$$T_{p_{1}}^{*} = \left[ \left( T_{p_{10}} + \frac{\frac{2\alpha F\rho_{p_{10}}}{3Cp_{r}} - T_{10}}{1 - \frac{Ap^{2}}{Cp_{r}}} \right) e^{-cz} + \frac{2\alpha F\rho_{p_{10}}}{3Ap^{2}} + \left( \frac{T_{10} - \frac{2\alpha F\rho_{p_{10}}}{3Ap^{2}}}{1 - \frac{Ap^{2}}{Cp_{r}}} \right) e^{-\frac{Ap^{2}z}{p_{r}}} \right] \frac{J_{1}(p)}{p}$$

Where 
$$\mathbf{u}_{1}^{*} = \int_{0}^{\infty} \mathbf{r} \, \mathbf{u}_{1} \, \mathbf{J}_{0} \left( pr \right) dr$$
 etc.

$$A = \frac{1}{(1-\phi)u_0}, E = \frac{u_{p_0} - u_0}{u_0}, C = \frac{2}{3p_r u_{p_0}}, F = \frac{T_{p_0} - T_0}{(1-\phi)u_0} \text{ And}$$

$$\rho_{p_1}^* = \rho_{p_0} \frac{J_1(p)}{p}$$

In which the Bessel functions of zero and first order, denoted as  $J_0$  and  $J_1$ , are located.

# **4.RESULTS AND DISCUSSION:**

Taking into account the following factors, numerical computations have been made.  $P_r = 0.72$ ,  $u_{10} = up_{10} = T_{10} = T_{10} = T_{10} = 0.1$ ,  $\phi = 0.01$ .



Figure 1: Perturbed Fluid Phase Temperature





Figure 1 shows the profiles of perturbation fluid phase temperature  $T_1$  and Figure 2 displays the profiles of perturbed particle phase temperature  $T_{p1}$  for  $\alpha = 0.2$  and for various values of Z, respectively. When r = 0, the fluid and particle temperatures are at their highest. It was also shown that, when a magnetic field is present, the temperature of the particles is less than that of the fluid.

Validation of results: Numerical computations are performed for both the velocity and temperature of fluid and particle phase in order to obtain the physical features of the problem and to discuss the results. It is found that the results of this study coincide with the results of B.k Rath [16]

#### CONCLUSION

In the incompressible dusty fluid, we have studied the temperature fluid flow of particles in a magnetic field and used the equation of heat in fluid phase and particle phase. The solution is obtained by solving linearized governing differential equation using Laplace transformation.

For different values of concentric parameter and different values of z, the particle temperature reduced.

#### REFERENCES

- Rath B.K, Mahapatro P.K, and Dash D.k, (2017): Effect of Volume Fraction along with concentration parameter in the dusty incompressible fluid published in Adv. Appl. Fluid. Mech. 20, No: 1, pp 117-12
- [2] Srivastava K.M. (2009): Effect of Hall current on the instability of an anisotropic plasma Jet.- J. Plasma Phys., vol.12, No.1, pp.33-43.
- [3] Panda T.C, Mishra, S.K, and Panda, K.C. (2001): Volume fraction and diffusion analysis in SPM modeling in an inertial frame of reference, Acta Ciencia Indica, XXVIIM, No. 4, 515.
- [4] Chen L, Gong J, Sun F, and Wu C, (2002): Heat transfer effect on the performance of MHD power plant. Energy Conversion and Management, vol.43, No.15, pp.2085-2095.
- [5] Purcell, E.M. (1978) the effect of fluid motions on the absorption of molecules by suspended particles, J. Fluid Mech. 84, 551-559.
- [6] Rath, B.K, Behera G.K, and Dash D.K, (2015): Solution of Longitudinal velocity of the fluid and the particle of he dustu fluid with the effect of volume fraction in the incompressible fluid of SPM. Adv. Appl. Fluid. Mech. 18, 155-162
- [7] Panda T, Mishra S.K, and Dash D.K, (2006): Modeling Dispersion of SPM in free convection flows in the vicinity of heated horizontal flat plate in Impact J. Sci. Tech. 1, 37-60.
- [8]Panda T, Mishra S.K, and Panda K.C,( 2016): Induced flow of suspended particulate matter (SPM) due to time dependent horizontally oscillating plate, Acta Ciencia Indica, Vol. 27M, (2), 233-239.
- [9]Rath, B.K., Behera, G.K., and Dash, D.K. (2015): Solution of Longitudinal velocity of the fluid and the particle of he dustu fluid with the effect of volume fraction in the incompressible fluid of SPM. Adv. Appl. Fluid. Mech. 18, 155-162
- [10]Panda T.C, Mishra S.K, and Panda K.C, (2001): Volume fraction and diffusion analysis in SPM modeling in an inertial frame of reference, Acta Ciencia Indica, XXVIIM, No. 4, 515.
- [11]Rat, B.K. and Ganesh, V. (2014); the longitudinal pretreated fluid velocity of the dusty fluid in the incompressible flow in cylindrical polar coordinates in Int. J. Res. Engg. Tech. 03, 769-772
- [12] Dash D.K, and Rath B.K, (2009): published in Far East J.Appl. Math, entitled "Effect of volume fraction in axi symmetric jet mixing of incompressible flow in cylindrical polar coordinates, 34(1), 83-94."
- [13] Panda, T.C., Mishra, S.K., and Dash D.K. (2006): Modeling Dispersion of SPM in free convection flows in the vicinity of heated horizontal flat plate in Impact J. Sci. Tech. 1, 37-60
- [14] Dutta N, & Das S.K, (1986): published in Note on axis-symmetric jet mixing of compressible dusty fluid, Acta Mechanica, 86, 103-109
- [15] Jha B.K, Apere C.A, (2010): Combined effects of Hall and ion-slip currents on unsteady MHD Couette flows in a rotating system. J. Phys. Soc. Japan, vol.79,
- [16] V. Ganesh, B. K. Rath, Sonal Bharti, D.K. Dash, (2020): numerical solution of longitudinal perturbation fluid and particle velocity on a dusty fluid jet compressible flow problem considering finite volume fraction JETIR September 2020, Volume 7, Issue 9 (ISSN-2349-5162).