Weighted Prakaamy Distribution: Properties, Applications and Analysis

D. Vedavathi Saraja[1*], C. Subramanian[2], N. Srinivasa Rao[3]

[1,2]*Department of Statistics, Annamalai University, Annamalainagar, Tamil Nadu, India; E-mail: sarajayoganand@gmail.com, E-mail: manistat@yahoo.co.in*

[3]*Department of Statistics, Andhra Loyola College, Vijayawada, India; E-mail: nunnasr@gmail.com*

Abstract: The paper introduces a new probability distribution, which is weighted version of the Prakaamy distribution. The paper explores various statistical properties of the Weighted Prakaamy (WP)distribution including probability density function (PDF), cumulative distribution function (CDF), moments, moment generating function, characteristics function, reliability analysis, ordered statistics, maximum likelihood estimation of parameters, entropies, likelihood ratiotest, and Bonferroni and Lorenz curves. The paper uses simulations to evaluate the performance of maximum likelihood estimators. The authorsapply the WP distribution to various real-life data sets from fields of engineering and medical science. This empirical analysis aims to evaluate the performance of the distribution in modeling and predicting real-world phenomena. The paper suggests that WP distribution outperforms other probability distributions including Prakaamy distribution, Exponential distribution, Erlang Truncated Exponential distribution, Power Lindley distribution and Lindley distribution.

Keywords: Weighted Distribution, Prakaamy Distribution, Reliability Analysis, Order Statistics, Maximum Likelihood Estimators.

1. INTRODUCTION

Weighted distributions are used in various fields, including biomedicine, reliability, ecology, and branching processes. They are crucial for modeling and analyzing lifetime data, especially when standard distributions are not suitable. Fisher introduced the concept of weighted distributions in 1934 to address ascertainment bias. Rao later (1965) developed this concept in a unified manner to handle situations where standard distributions were inadequate. Weighted distributions allow observations to be recorded according to some weighted function. They reduce to length-biased distributions when the weight function considers only the length of the units. The concept of length-bias was first introduced by Cox [1] and Zelen [11]. Lappi and Bailey [3], used weighted distributions to analyze the HPS diameter increment data. In fisheries, Taillie *et al* [9] modeled populations of fish stock using weights. Dey et al (2015) discussed weighted exponential distribution with its properties and different methods of estimation. Kilany (2016) have obtained the weighted version of lomax distribution. Recently Shanker & Shukla [7] discussed a new generalized size-biased, Poisson-Lindley distribution with its application to model size distribution. Also, Rather and Subramanian (2018) discussed the characterization and estimation of length biased weighted generalized uniform distribution. Rather et al (2018) obtained a new size biased Ailamujia distribution with applications in engineering and medical science which shows more flexibility than classical distributions. Subramanian and Rather [8] obtained the weighted version of exponentiated mukherjee-islam distribution with statistical properties. Rather and Subramanian (2018) have discussed the statistical properties and applications of length biased sushila distribution. Rather and Subramanian [6] discussed on weighted sushila distribution with properties and application which shows more flexibility then the subject distribution. Recently, Ganaie, Rajagopalan and Rather (2019), discussed the weighted Aradhana distribution properties and applications. Rather and Ozel (2020), discussed on weighted power lindley distribution with applications on the life time data which shows more flexibility than the classical distribution. Overall, weighted distributions provide a flexible framework for handling data that does not conform to standard probability distributions, making them valuable tools in various scientific disciplines. Researchers have explored these distributions and their properties extensively, contributing to a deeper understanding of their applications. Various weighted probability models have been developed. Some examples include the weighted exponential distribution, weighted Lomax distribution, weighted modified Weibull distribution, length-biased weighted generalized uniform distribution, and size-biased Ailamujia distribution.

2. WEIGHTED PRAKAAMY DISTRIBUTION

The probability density function (PDF) of Prakaamy distribution is given by

$$g(y;\theta)=\frac{\theta^6}{\theta^5+120}\left(1+y^5\right)e^{-\theta y} \qquad ; y>0, \theta>0 \tag{1}$$

The corresponding cumulative distribution function (CDF) of Prakaamy distribution is

$$G(y;\theta)=1-\left(1+\frac{\theta y\left(\theta^4 y^4+5\theta^3 y^3+20\theta^2 y^2+60\theta y+120\right)}{\theta^5+120}\right)e^{-\theta y} \qquad y>0, \theta>0 \tag{2}$$

Now,Suppose *Y* is a non-negative random variable with PDF $g(y;\theta)$. Let *w(y)*= y^c be the non-negative weight function, then the PDF of Weighted Prakaamy distribution is given by $f(y;\theta,c)=\frac{y^c g(y;\theta)}{E(y^c)}$ (3)

Now

$$E(y^c)=\int_0^\infty y^c g(y;\theta)\,dy$$

$$E(y^c)=\int_0^\infty y^c \frac{\theta^6}{\theta^5+120}\left(1+y^5\right)e^{-\theta y}\,dy$$

$$E(y^c)=\frac{\theta^6}{\theta^5+120}\left(\int_0^\infty e^{-\theta y}y^c dy+\int_0^\infty e^{-\theta y}y^{c+5}dy\right)$$

$$E(y^c)=\frac{\theta^6}{\theta^5+120}\left(\frac{\Gamma(c+1)}{\theta^{c+1}}+\frac{\Gamma(c+6)}{\theta^{c+6}}\right)$$

After simplification we get

$$E(y^c)=\frac{\theta^6}{\theta^5+120}\frac{c!}{\theta^{c+1}}\left(1+\frac{(c+5)(c+4)(c+3)(c+2)(c+1)}{\theta^5}\right)$$

$$E(y^c)=\frac{\theta^6}{\theta^5+120}\frac{c!}{\theta^{c+6}}\left(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1)\right)$$

$$E(y^c)=\frac{c!}{(\theta^5+120)\,\theta^c}\left(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1)\right) \tag{4}$$

Substituting equation (01) and (04) in (03) we get

$$f(y;\theta,c)=\frac{\theta^{c+6}y^c(1+y^5)\,e^{-\theta y}}{c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))} \tag{5}$$

The corresponding CDF of Weighted Prakaamy distribution is

$$F(y;\theta,c)=\int_0^y \frac{\theta^{c+6}y^c(1+y^5)e^{-\theta y}}{c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))}dy$$

$$F(y;\theta,c)=\frac{\theta^{c+6}}{c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))}\left(\int_0^y e^{-\theta y}y^c dy+\int_0^y e^{-\theta y}y^{c+5}dy\right)$$

After simplification we get

$$F(y;\theta,c)=\frac{\theta^5\gamma\big((c+1),\theta y\big)+\gamma\big((c+6),\theta y\big)}{c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))} \tag{6}$$

3. MOMENTS OF WEIGHTED PRAKAAMY DISTRIBUTION

Let Y denotes the random variable following Weighted Prakaamy distribution with parameters c and θ, then the rth raw moment about origin of the Weighted Prakaamy distribution is given by

$$\mu_r'=\int_0^\infty y^r f(y;\theta,c)dy$$

$$\mu_r'=\int_0^\infty y^r \frac{\theta^{c+6}y^c(1+y^5)e^{-\theta y}}{c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))}dy$$

After simplification we get

$$\mu_r'=\frac{(r+c)!\big(\theta^5+(r+c+5)(r+c+4)(r+c+3)(r+c+2)(r+c+1)\big)}{\theta^{r+1}c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))}$$

4. MOMENT GENERATING FUNCTION AND CHARACTERISTICS FUNCTION

The moment generating function of Weighted Prakaamy distribution is given by

$$M_{Yw}(t)=\int_0^\infty e^{ty} f(y;\theta,c)\,dy$$

$$M_{Y_w}(t)=\int_0^\infty e^{tx}\frac{\theta^{c+6}y^c(1+y^5)e^{-\theta y}}{c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))}dy$$

After simplification we get

$$M_{Y_w}(t) = \sum_{j=0}^{\infty} \frac{(t)^j}{j!} \frac{(j+c)!\left(\theta^5 + (j+c+5)(j+c+4)(j+c+3)(j+c+2)(j+c+1)\right)}{\theta^{j+1} c!\left(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1)\right)}$$

Also, the characteristics function of Weighted Prakaamy distribution is given by

$$\phi_{Y_w}(t) = \sum_{j=0}^{\infty} \frac{(\iota t)^j}{j!} \frac{(j+c)!\left(\theta^5 + (j+c+5)(j+c+4)(j+c+3)(j+c+2)(j+c+1)\right)}{\theta^{j+1} c!\left(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1)\right)}$$

5. THE RELIABILITY ANALYSIS

In this section, we have obtained the survival function, hazard rate and Reverse hazard rate function of the proposed Weighted Prakaamy distribution.

5.1. Survival Function

The survival function is defined as the probability that a system survives beyond a specified time. It is also known as reliability function and can be computed as complement of the cumulative distribution function of the model. The reliability function or the survival function of Weighted Prakaamy distribution can be computed as

$$S(y_w) = 1 - F(y:\theta, c)$$

$$S(y_w) = 1 - \frac{\theta^5 \gamma\left((c+1), \theta y\right) + \gamma\left((c+6), \theta y\right)}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}$$

5.2. Hazard Function

The hazard function, also known as hazard rate, is defined as the instantaneous failure rate or force of mortality and is given by

$$h(y_w) = \frac{f(y;\theta,c)}{1 - F(y;\theta,c)}$$

$$h(y_w) = \frac{\left(\dfrac{\theta^{c+6} y^c (1+y^5) e^{-\theta y}}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)}{\left(1 - \dfrac{\theta^5 \gamma\left((c+1), \theta y\right) + \gamma\left((c+6), \theta y\right)}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)}$$

$$h(y_w) = \frac{\left(\theta^{c+6} y^c (1+y^5) e^{-\theta y}\right)}{\left(c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1)) - (\theta^5 \gamma\left((c+1), \theta y\right) + \gamma\left((c+6), \theta y\right))\right)}$$

5.3. Reverse Hazard Function

The reverse hazard function of Weighted Prakaamy distribution is given by

$$h^r(y_w) = \frac{f(y;\theta,c)}{F(y;\theta,c)}$$

$$h^r(y_w) = \frac{\left(\dfrac{\theta^{c+6} y^c (1+y^5) e^{-\theta y}}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)}{\left(\dfrac{\theta^5 \gamma((c+1),\theta y) + \gamma((c+6),\theta y)}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)}$$

$$h^r(y_w) = \frac{\theta^{c+6} y^c (1+y^5) e^{-\theta y}}{\theta^5 \gamma((c+1),\theta y) + \gamma((c+6),\theta y)}$$

Graphical representation of pdf plot is shown in Figure 1 and Figure 2, Figure3 and Figure 4 shows cdf plot, Figure 5 and Figure 6 shows the survival function and Figure 7 and Figure 8 shows the hazard rate.

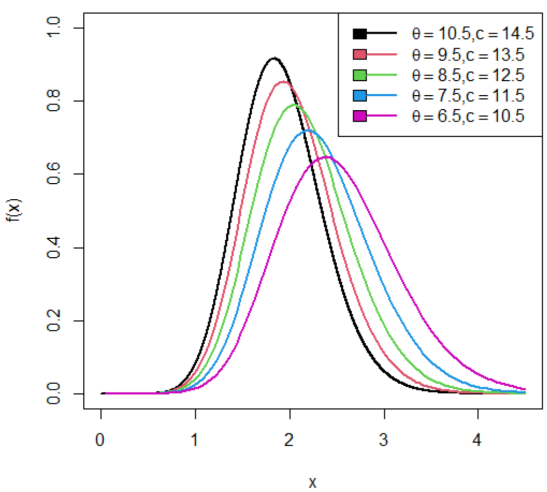


Figure 1: PDF plot of Weighted Prakaamy distribution

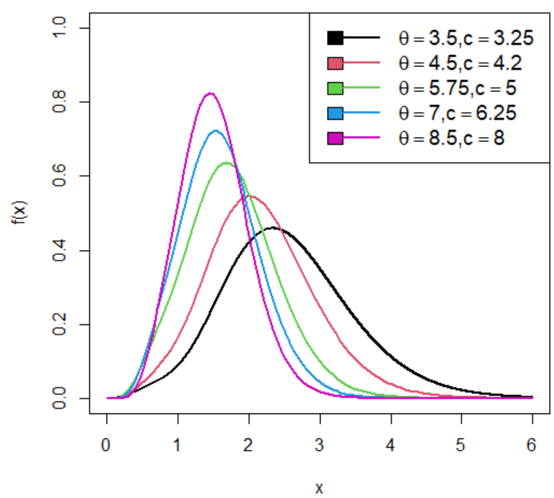


Figure 2: PDF plot of Weighted Prakaamy distribution

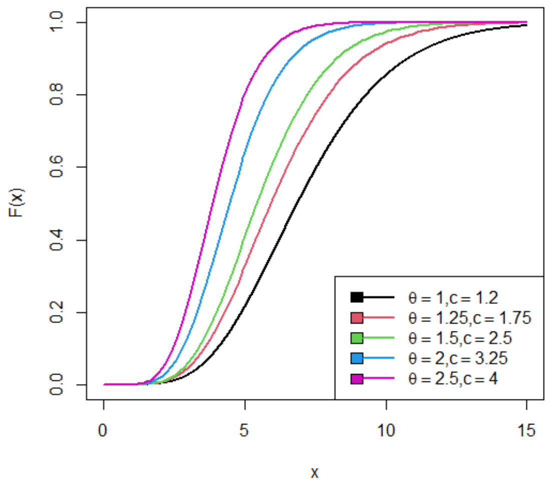


Figure 3: CDF plot of Weighted Prakaamy distribution

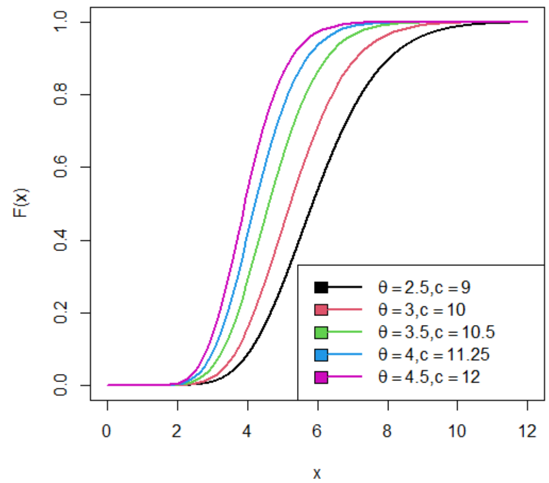


Figure 4: CDF plot of Weighted Prakaamy distribution

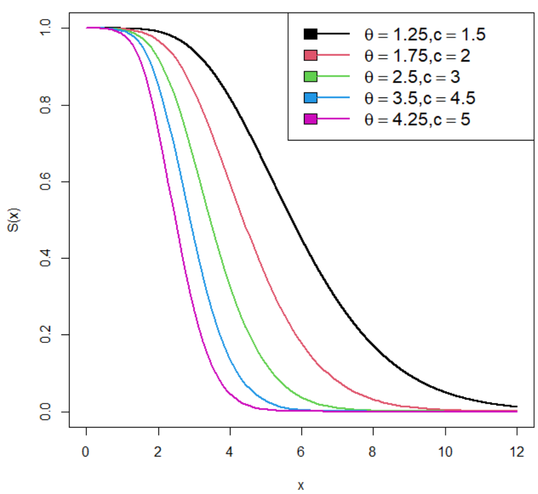


Figure 5: Plot of survival function of Weighted Prakaamy distribution

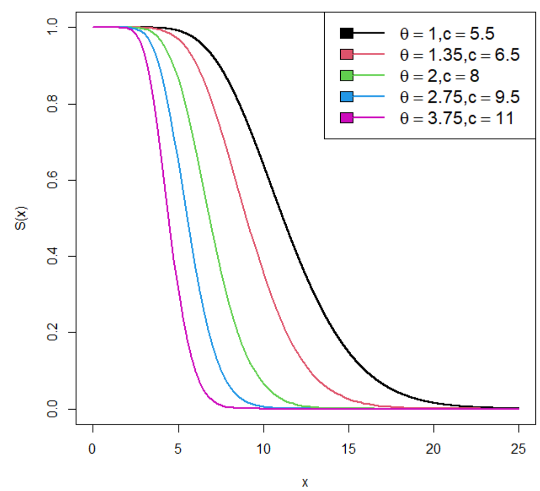


Figure 6: Plot of survival function of Weighted Prakaamy distribution

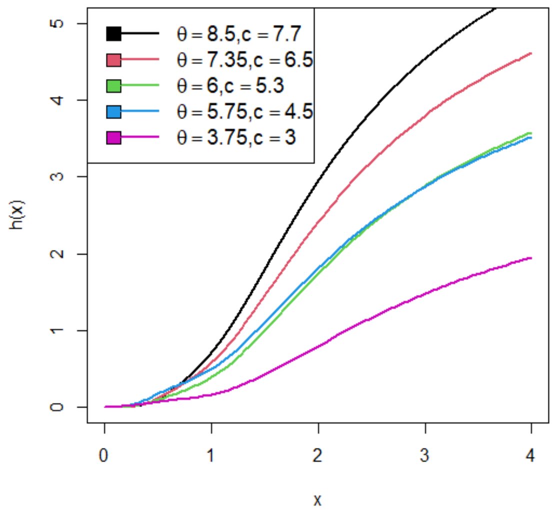


Figure 7: Plot of hazard function of Weighted Prakaamy distribution

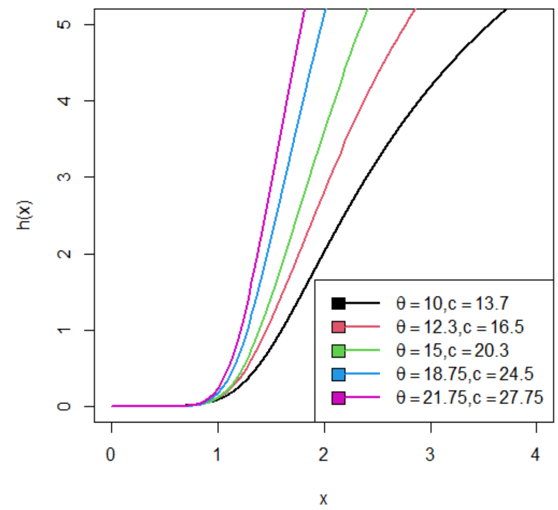


Figure 8: Plot of hazard function of Weighted Prakaamy distribution

6. Ordered Statistics

Let Y_1, Y_2, Y_3, Y_4, … , Y_n be a random sample of size '*n*' drawn from a given population following Weighted Prakaamy distribution . Then the ordered statistics associated with the given sample is given by

$Y_{(1)} \leq Y_{(2)} \leq Y_{(3)} \leq \ldots \leq Y_{(n)}$

Where,

$Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_n)$

And

$Y_{(n)} = \max(Y_1, Y_2, \ldots\ldots Y_n)$

Now, the PDF of *rth* ordered statistics $Y_{(r)}$ is

$$f_{Y(r)}(y) = \frac{n!}{(r-1)!\,(n-r)!} f(y)\left(F(y)\right)^{r-1}\left(1-F(y)\right)^{n-r}$$

$$f_{Y(r)}(y) = \frac{n!}{(r-1)!\,(n-r)!}\left(\frac{\theta^{c+6} y^c (1+y^5) e^{-\theta y}}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)$$
$$\times\left(\frac{\theta^5 \gamma((c+1),\theta y) + \gamma((c+6),\theta y)}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)^{r-1}$$
$$\times\left(1 - \frac{\theta^5 \gamma((c+1),\theta y) + \gamma((c+6),\theta y)}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)^{n-r}$$

Therefore, the PDF of highest ordered statistics $Y_{(n)}$ is

$$f_{Y(r)}(y) = n\left(\frac{\theta^{c+6} y^c (1+y^5) e^{-\theta y}}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)\left(\frac{\theta^5 \gamma((c+1),\theta y) + \gamma((c+6),\theta y)}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)^{n-1}$$

And the PDF of first ordered statistics $Y_{(1)}$is

$$f_{Y(r)}(y) = n\left(\frac{\theta^{c+6} y^c (1+y^5) e^{-\theta y}}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)\left(1-\frac{\theta^5 \gamma((c+1),\theta y) + \gamma((c+6),\theta y)}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)^{n-r}$$

7. ESTIMATION

Maximum likelihood estimation method is one of the most useful method for estimating the different parameters of the distribution. Let Y_1 , Y_2 , Y_3 , Y_4 , … , $Y_n$be the random sample of size n drawn from the Weighted Prakaamy distribution. Then the likelihood function of the given random sample is given by

$$L(\theta;c) = \prod_{i=1}^{n} f(y_i;\theta,c)$$

$$L(\theta;c) = \prod_{i=1}^{n} \frac{\theta^{c+6} {y_i}^c (1+{y_i}^5) e^{-\theta y_i}}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}$$

$$L(\theta;c) = \left(\frac{\theta^{c+6}}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)^n \prod_{i=1}^{n} {y_i}^c (1+{y_i}^5) e^{-\theta y_i}$$

Applying log on both sides, we get

$$\log L(\theta;c) = \log\left(\left(\frac{\theta^{c+6}}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)^n \prod_{i=1}^{n} {y_i}^c (1+{y_i}^5) e^{-\theta y_i}\right)$$

$$\log(L(\theta;c)) = n(c+6)\log(\theta) - n\log(c!) - n\log\left(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1)\right) + \left(\sum_{i=1}^{n} c\log y_i + \sum_{i=1}^{n}\log(1+{y_i}^5)\right) + \left(-\theta\sum_{i=1}^{n} y_i\right) \quad (7)$$

Differentiating equation (07) partially with respect to θ and equating to zero we get we get

$$\frac{n(c+6)}{\theta} - \frac{5\theta^4 n}{\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1)} - \sum_{i=1}^{n} y_i = 0$$

$$\frac{n(c+6)}{\theta}-\frac{5\theta^{4}n}{\theta^{5}+(c+5)(c+4)(c+3)(c+2)(c+1)}-\sum_{i=1}^{n}y_i=0 \quad (8)$$

Differentiating equation (07) partially with respect to c and equating to zero we get

$$n\log\theta-n\Psi(c+1)-\frac{n\Phi(c)}{\theta^{5}+(c+5)(c+4)(c+3)(c+2)(c+1)}+\sum_{i=1}^{n}\log y_i=0 \quad (9)$$

Where,

$$\Psi(c+1)=\frac{d}{dc}\log c!=\frac{1}{c!}\frac{d}{dc}c!$$

is called the digamma function.

And

$$\Phi(c)=\frac{d}{dc}(c+5)(c+4)(c+3)(c+2)(c+1)$$

is the function of c

By solving equation (08) and (09) we get the maximum likelihood estimators of the parameters of Weighted Prakaamy distribution. Since it is very complicated to estimate *θandc, so* we use mathematica or Newton Raphson method.

8. ENTROPY

The concept of entropy is a fundamental and versatile concept with applications in various fields, including probability and statistics, physics, communication theory, and economics. Entropy measures play a crucial role in quantifying the diversity, uncertainty, and randomness within systems. In particular, the entropy of a random variable Y serves as a metric for assessing the degree of uncertainty or variation associated with it.

8.1. Renyi entropy

The Rényi entropy is indeed significant in the fields of ecology and statistics, particularly as an index of diversity. It was introduced by Alfréd Rényi in 1957.

The Renyi entropy is defined as

$$e(\beta)=\frac{1}{1-\beta}\log\left(\int_0^\infty (f(y;\theta,c))^\beta\,dy\right)$$

$$e(\beta)=\frac{1}{1-\beta}\log\left(\int_0^\infty\left(\frac{\theta^{c+6}y^c(1+y^5)e^{-\theta y}}{c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))}\right)^\beta\right)$$

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\theta^{c+6}}{c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))}\right)^\beta\int_0^\infty y^{\beta c}\left(1+y^5\right)^\beta e^{-\beta\theta y}dy\right)$$

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\theta^{c+6}}{c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))}\right)^\beta\left(\int_0^\infty y^{\beta c}\left(1+y^5\right)^\beta e^{-\beta\theta y}dy\right)\right)$$

After simplification we get

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\theta^{c+6}}{c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))}\right)^\beta\left(\sum_{k=0}^{\beta}{}^{\beta}C_k\,\frac{\Gamma(\beta c+5k+1)}{(\beta\theta)^{\beta c+5k+1}}\right)\right)$$

8.2. Tsallis Entropy

Tsallis introduced a mathematical expression for Tsallis entropy in 1988 and is particularly useful for systems that exhibit non-extensive properties, such as long-range interactions, self-organization, and non-Gaussian statistics.

The Tsallis entropy is given by

$$S_\lambda=\frac{1}{\lambda-1}\left(1-\int_0^\infty\left(f(y;\theta,c)\right)^\lambda dy\right)$$

$$S_\lambda=\frac{1}{\lambda-1}\left(1-\int_0^\infty\left(\left(\frac{\theta^{c+6}y^c(1+y^5)e^{-\theta y}}{c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))}\right)\right)^\lambda dy\right)$$

$$S_\lambda=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^{c+6}}{c!(\theta^5+(c+5)(c+4)(c+3)(c+2)(c+1))}\right)^\lambda\int_0^\infty\left(y^c(1+y^5)e^{-\theta y}\right)^\lambda dy\right)$$

After simplification we get

$$S_\lambda = \frac{1}{\lambda - 1}\left(1 - \left(\frac{\theta^{c+6}}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)^\lambda \left(\sum_{k=0}^{\lambda} {}^\lambda C_k \frac{\Gamma(\lambda c + 5k + 1)}{(\lambda\theta)^{\lambda c + 5k + 1}}\right)\right)$$

9. LIKELIHOOD RATIO TEST

Let Y_1, Y_2, Y_3, Y_4, … , Y_n be a random sample of size '*n*' drawn from a given population following Weighted Prakaamy distribution .

$$H_0 : f(y) = g(y;\theta) \qquad against \qquad H_1 : f(y) = f(y;\theta, c)$$

In order to test whether the random sample of size *n* comes from the Prakaamy distribution or Weighted Prakaamy distribution, then following the likelihood ratio test statistic is used

$$LR = \frac{L_0}{L_1} = \prod_{i=1}^{n} \frac{g(y_i;\theta)}{f(y_i;\theta, c)}$$

$$LR = \prod_{i=1}^{n}\left(\frac{\frac{\theta^6}{\theta^5 + 120}\left(1 + y^5\right)e^{-\theta y}}{\frac{\theta^{c+6} y^c (1 + y^5) e^{-\theta y}}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}}\right)$$

$$LR = \left(\frac{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}{(\theta^5 + 120)\theta^c}\right)^n \prod_{i=1}^{n}\left(\frac{1}{y_i}\right)^c$$

We reject the null hypothesis if the likelihood ratio is small i.e.,

$$LR \le k$$

Where *k* is a constant such that

$$P(LR \le k) = \alpha$$

$$\alpha = P\left(\left(\frac{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}{(\theta^5 + 120)\theta^c}\right)^n \prod_{i=1}^{n}\left(\frac{1}{y_i}\right)^c \le k\right)$$

$$\alpha = P\left(\prod_{i=1}^{n}\left(\frac{1}{y_i}\right)^c \le k\left(\frac{(\theta^5 + 120)\theta^c}{c!(\theta^5 + (c+5)(c+4)(c+3)(c+2)(c+1))}\right)^n\right)$$

10. BONFERRONI AND LORENZ CURVES

The Bonferroni and Lorenz curves are versatile tools used in economics, statistics, reliability, medicine, insurance, and demography. The Bonferroni correction is employed to control Type I errors in statistical analysis. Lorenz curves help visualize income and wealth distribution and are vital for assessing social inequalities in various fields. These curves provide valuable insights, making them essential in decision-making across a range of disciplines.

10.1. Bonferroni Curves

The Bonferroni curve is given by

$$B(p)=\frac{1}{p\,\mu_1'}\int_0^q y\,f(y;\theta,c)\,dy$$

Where $q=F^{-1}(p)$ and

μ_1' *is the mean of Weighted* Pr*akamy distribution*

$$B(p)=\frac{1}{p\,\mu_1'}\int_0^q y\frac{\theta^{c+6}y^{c}(1+y^{5})e^{-\theta y}}{c!(\theta^{5}+(c+5)(c+4)(c+3)(c+2)(c+1))}dy$$

$$B(p)=\frac{1}{p\,\mu_1'}\frac{\theta^{c+6}}{c!(\theta^{5}+(c+5)(c+4)(c+3)(c+2)(c+1))}\int_0^q y^{c+1}(1+y^{5})e^{-\theta y}dy$$

$$B(p)=\frac{1}{p\,\mu_1'}\frac{\theta^{c+6}}{c!(\theta^{5}+(c+5)(c+4)(c+3)(c+2)(c+1))}\left(\int_0^q e^{-\theta y}y^{c+1}dy+\int_0^q e^{-\theta y}y^{c+5}dy\right)$$

After simplification we get

$$B(p)=\frac{1}{p\,\mu_1'}\frac{\theta^{c+6}}{c!(\theta^{5}+(c+5)(c+4)(c+3)(c+2)(c+1))}\left(\frac{1}{\theta^{c+2}}\gamma\big((c+2),\theta q\big)+\frac{1}{\theta^{c+7}}\gamma\big((c+7),\theta q\big)\right)$$

10.2. Lorenz Curves

The Lorenz curve is given by

$$L(p)=p\,B(p)$$

$$L(p)=\frac{1}{\mu_1'}\frac{\theta^{c+6}}{c!(\theta^{5}+(c+5)(c+4)(c+3)(c+2)(c+1))}\left(\frac{1}{\theta^{c+2}}\gamma\big((c+2),\theta q\big)+\frac{1}{\theta^{c+7}}\gamma\big((c+7),\theta q\big)\right)$$

11. SIMULATION

Simulations provide a comprehensive and versatile approach to understand the behavior of maximum likelihood estimators across different sample sizes. This knowledge can guide better dicision making, minimize risks, and improve the reliability and efficiency of statistical analysis in various fields such as finance, healthcare, engineering etc. Simulations enable us to anticipate how MLEs will behave under a wide array of sample sizes, even those not easily attainable in practice. This predictive capacity assists in understanding how the estimator's bias, variance and efficiency will change as sample size fluctuates. Simulations can help in identifying the optimal sample size for an MLE application. We have studied the performance of ML estimators for different sample sizes (n= 25, 50, 75, 100, 200, 300). The inverse CDF technique was employed for data simulation in the R-software and the process was repeated 700 times to calculate bias, variance and mean squared error (MSE). From table 1, it is noted that for different values of parameters with different sample sizes of Weighted Prakaamy distribution, the decreasing trend has been observed in variance, bias and MSE as the sample size increases. The decreasing bias suggests that the ML estimation tend to approach the true parameter values as the sample size increases. The decreasing variance implies that the estimators become more precise and stable with larger sample sizes, as they exhibit less variability across repeated simulations. Further, Figure 9, Figure 10, Figure 11,Figure 12, Figure 13, and Figure 14 shows the histogram of simulation on different values of the parameters with different the sample size. Consequently, MSE which combines the bias and variance also decreases as the sample size an increase, indicating improved overall estimation accuracy.This result indicates that the performance of ML estimators improves consistently with larger sample sizes in Weighted Prakaamy distribution.

Table 1: Estimation of Bias, Variance and MSE for different sample sizes

n	θ=3			c=2		
	Bias	**Variance**	**MSE**	**Bias**	**Variance**	**MSE**
25	0.8162067	0.4970276	1.163221	1.92244	2.731128	6.426904
50	0.5163385	0.1357713	0.4023768	1.241929	1.12538	2.667767
75	0.2770153	0.1298997	0.2066372	0.7043924	0.8085554	1.304724
100	0.1135018	0.07648831	0.08937097	0.3129216	0.2885902	0.3865101
200	0.0778941	0.07055497	0.07662246	0.1551423	0.2694947	0.2935638
300	0.006648184	0.01436286	0.01440706	0.002428027	0.09489351	0.0948994
	θ=7			c=5		
	Bias	**Variance**	**MSE**	**Bias**	**Variance**	**MSE**
25	1.558031	3.038257	5.465718	1.93421	4.083858	7.825028
50	0.7976739	1.632195	2.268479	1.058869	2.648107	3.769311
75	0.5962892	0.8862764	1.241837	0.9493708	1.54787	2.449175
100	0.2388727	0.3003692	0.3574294	0.2393312	0.4331804	0.4904598
200	0.04890744	0.2667191	0.2691111	0.07504574	0.3539969	0.3596288
300	-0.00287561	0.1361568	0.136165	-0.0572108	0.1670096	0.1702826
	θ=4			c=9		
	Bias	**Variance**	**MSE**	**Bias**	**Variance**	**MSE**

25	0.6303604	0.8315047	1.228859	2.075204	10.28468	14.59115
50	0.3254438	0.5643284	0.6702421	1.187516	8.580748	9.990943
75	0.2803734	0.5540099	0.6326191	1.101834	6.80588	8.019918
100	0.2355682	0.3986065	0.4540988	0.7299452	4.902112	5.434932
200	0.08571555	0.1677978	0.1751449	0.3366784	2.377541	2.490893
300	0.002031912	0.1086348	0.1086389	-0.02944885	1.253967	1.254834
	θ=6			***c*=11**		
	Bias	**Variance**	**MSE**	**Bias**	**Variance**	**MSE**
25	2.171407	2.735779	7.450785	5.966887	21.64485	57.24859
50	0.6245353	1.02498	1.415025	2.187509	9.07805	13.86324
75	0.4367386	1.038073	1.228814	1.23183	7.104936	8.62234
100	0.4492844	0.5627282	0.7645847	1.117345	3.47835	4.72681
200	0.3041986	0.3089941	0.4015309	0.943277	2.396857	3.286629
300	0.1114169	0.1137174	0.1261311	0.2377907	0.8707666	0.927311

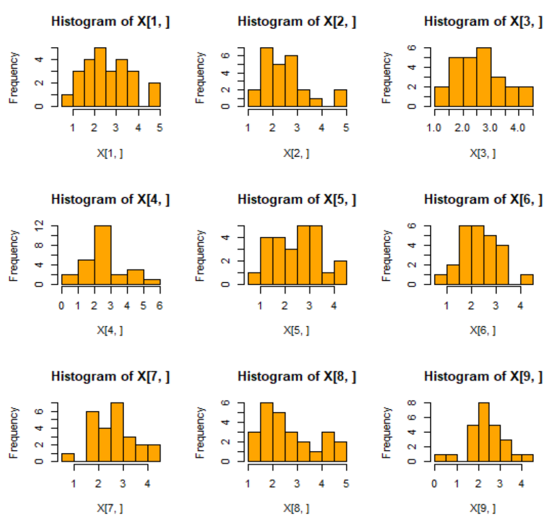


Figure 9: Histogram of simulation when *n*=25,

θ= 3, *c*=2

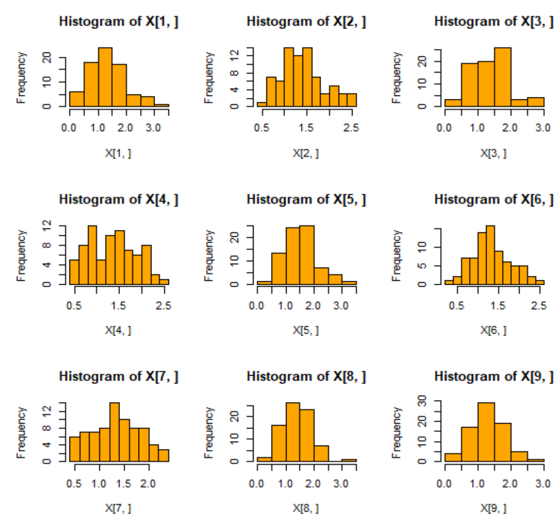


Figure 10: Histogram of simulation when *n*=75,

θ= 7, c=5

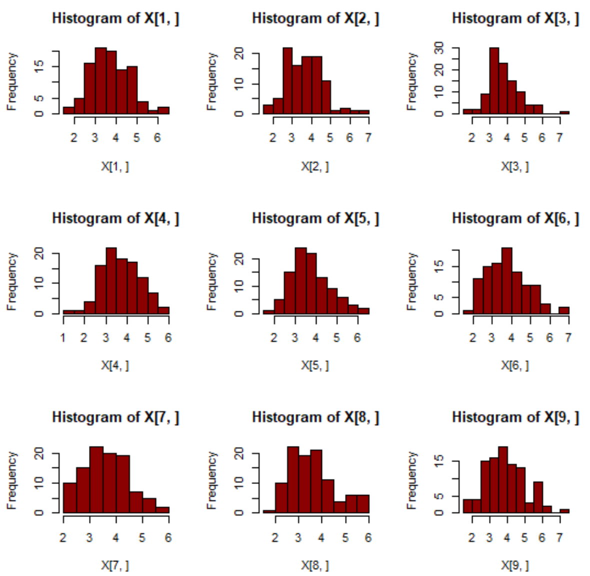


Figure 11: Histogram of simulation when n=100, θ= 4, c=9

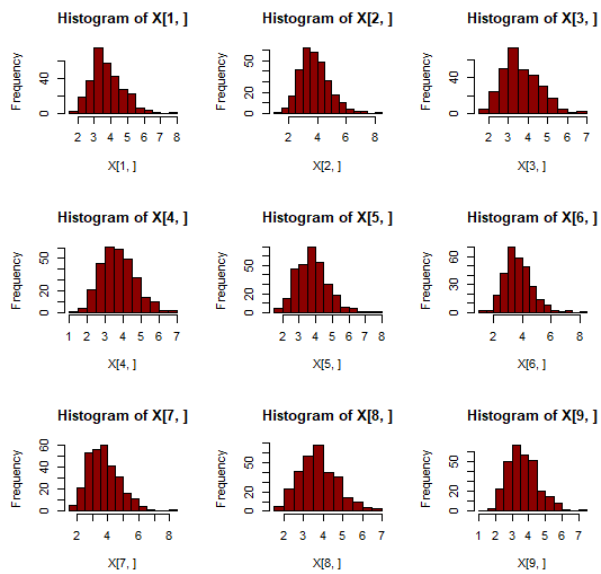


Figure 12: Histogram of simulation when n=300, θ= 4, c=9

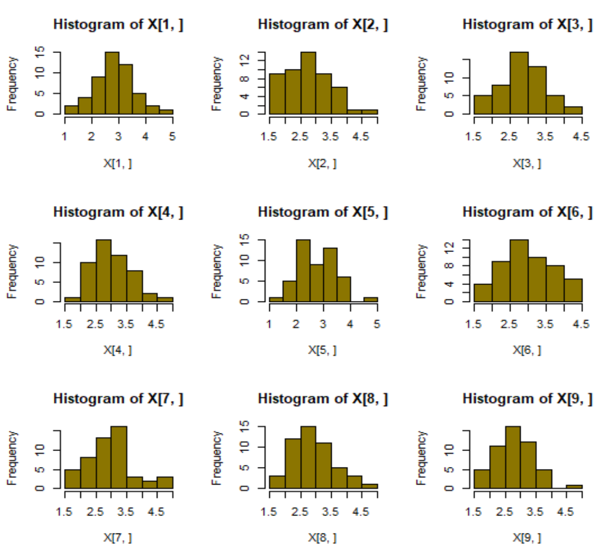


Figure 13: Histogram of simulation when n=50, θ= 6, c=11

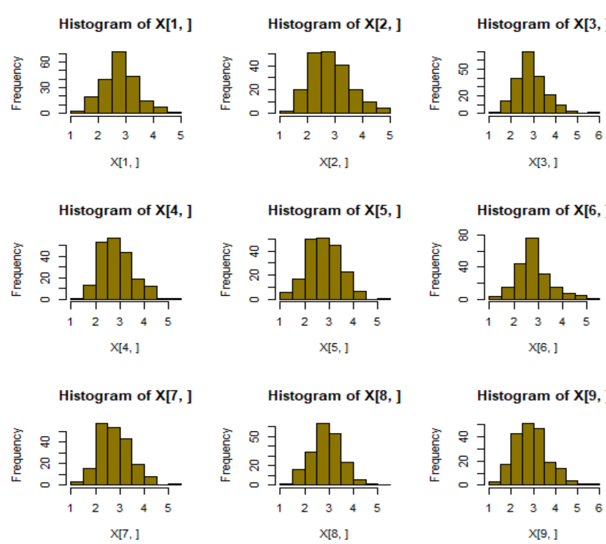


Figure 14: Histogram of simulation when n=200, θ= 6, c=11

12. APPLICATION

In this section, we use and analyse the two real-life data sets to show that the Weighted Prakaamy distribution fits better than the Prakaamy distribution, Exponential distribution, Erlang Truncated Exponential distribution, Power Lindley distribution and Lindley distribution. The following two data sets are provided below as

Data set 1: The data set is the strength data of glass of the aircraft window reported by Fuller, et al. [14] and are given as

18.83, 20.80, 21.657, 23.03, 23.23, 24.05, 24.321, 25.50, 25.52, 25.80, 26.69, 26.77, 26.78, 27.05, 27.67, 29.90, 31.11, 33.20, 33.73, 33.76, 33.89, 34.76, 35.75, 35.91, 36.98, 37.08, 37.09, 39.58, 44.045, 45.29, 45.431

Data set 2: Relief in minute's analgesic data of 20 patients has been reported by Gross and Clark in (1975).

1.1, 1.4, 1.3, 1.7, 1.9, 1.8, 1.6, 2.2, 1.7, 2.7, 4.1, 1.8, 1.5, 1.2, 1.4, 3.0, 1.7, 2.3, 1.6, 2.0

Estimates of the unknown parameters are carried out in R software along with calculation of model comparison criterion values like AIC, AICC, BIC and *-2logL*values. In order to compare the two models, the AIC (Akaike information criterion), AICC (corrected Akaike information criterion) and BIC (Bayesian information criterion) are used. The better distribution corresponds to lesser AIC, AICC and BIC values. The generic formulas for calculation of AIC, AICC and BIC are

$$AIC = 2k - 2\log L; \quad AICC = AIC + \frac{2k(k+1)}{n-k-1} \quad \text{and} \quad BIC = k\log n - 2\log L$$

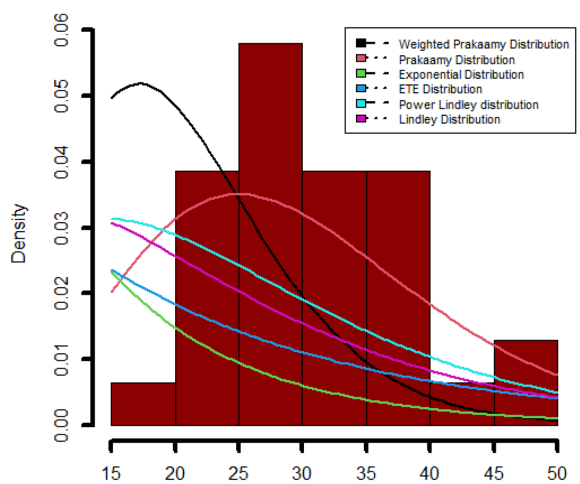
where k is the number of parameters in the statistical model, n is the sample size and $-LogL$ is the maximized value of the log-likelihood function under the considered model. Table 2 shows the parameter estimation and standard error values. Table 3 shows the performance of the distributions. Figue 15 shows Fitting density curves of data set 1 based on glass strength of air craft windows and Figure 16 Fitting density curves of data set 2 based on relief in minutes of analgesic patients.

Table 2: Shows values of ML estimates, and corresponding standard errors

Data Set	Distribution	Parameter	MLE	Standard Error
1	Weighted Prakaamy	θ	0.6140409	0.1566656
		c	12.9204417	4.7637234
	Prakaamy	θ	0.19472415	0.01427744
	Exponential	θ	0.032459933	0.005824437
	Erlang truncated Exponential	β	0.1421420	2.4169678
		θ	0.2591881	5.0312330
	Power Lindley	β	0.1421420	2.4169678
		θ	0.2591881	5.0312330
	Lindley	θ	0.062989929	0.008004739
2	Weighted Prakaamy	θ	6.506044	1.556488
		c	6.926375	2.743594
	Prakaamy	θ	2.273508	0.161589
	Exponential	θ	0.5263164	0.1176875
	Erlang truncated Exponential	β	0.8244015	74.8236370
		θ	1.0172774	160.2527526
	Power Lindley	β	2.2529461	0.3067604
		θ	0.3444791	0.0996840
	Lindley	θ	0.8161188	0.1360929

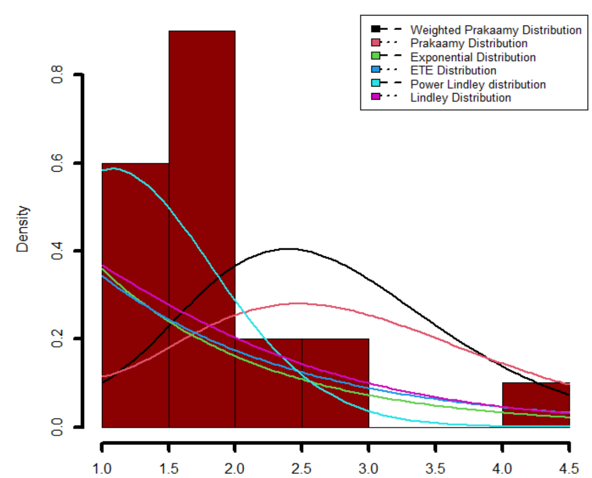
Table 3: Shows values of 2logL, AIC, BIC, and AICC

Data Set	Distribution	-2logL	AIC	BIC	AICC
1	**Weighted Prakaamy**	208.2532	212.2532	215.1211	212.6817714
	Prakaamy	223.0869	225.0869	226.5209	225.224831
	Exponential	274.5321	276.5321	277.9661	276.670031
	Erlang truncated Exponential	274.5321	278.5321	281.4001	278.9606714
	Power Lindley	274.5321	278.5321	281.4001	278.9606714
	Lindley	253.9925	255.9925	257.4265	256.130431
2	**Weighted Prakaamy**	37.63999	41.63999	43.63146	42.34587235
	Prakaamy	61.43961	63.43961	64.43534	63.66183222
	Exponential	65.67416	67.67416	68.66989	67.89638222
	Erlang truncated Exponential	65.67416	69.67416	71.66562	70.38004235
	Power Lindley	40.86396	44.86396	46.85543	45.56984235
	Lindley	60.4991	62.4991	63.49483	62.72132222



Glass strength of the air craft window

Figure 15: Fitting density curves of data set 1 based on glass strength of air craft windows



Relief in minutes of analgesic patients

Figure 16: Fitting density curves of data set 2 based on relief in minutes of analgesic patients

From Table 3, it has been observed that the Weighted Prakaamy distribution has smaller AIC, AICC, -LogL and BIC values as compared to the Prakaamy distribution, Exponential distribution, Erlang Truncated Exponential distribution, Power Lindley distribution and Lindley distribution, which clearly indicates that Weighted Prakaamy distribution fits better than Prakaamy distribution, Exponential distribution, Erlang Truncated Exponential distribution, Power Lindley distribution and Lindley distribution. Hence, we can conclude that the Weighted Prakaamy distribution leads to a better fit than the above other distributions.

CONCLUSIONS

In the present study we have studied a Weighted Prakaamy distribution as a new generalization of Prakaamy distribution. The new distribution is generated by using the weighting technique and taking the one parameter Prakaamy distribution as the base distribution. Some mathematical properties along with reliability measures are discussed. The hazard rate function and reliability behaviour of the Weighted Prakaamy distribution exhibits that subject distribution can be used as a lifetime model. Finally real life data has been analysed for coimparison purpose, and it has been analysed that weighted prakaamy distribution shows better performance than Prakaamy, Exponential, Erlang truncated Exponential, Power Lindley and Lindley distributions.

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